Key

Outcome D Study Guide

One Solution, No Solution, Infinite Many Solutions

*Determine the type of solution

$\cancel{5X} + 6 = 5X$
$-\cancel{5x} \quad -5x$
$6 = 0$
$\boxed{\text{No Solution}}$

$-3X + 10 = 2X - 5X + 10$
$-3x + 10 = -3x + 10$
$+3x \quad +3x$
$10 = 10$
$\boxed{\text{Many solutions}}$

Solving Systems using Substitution

*Isolate one variable, and substitute into the other equation

$X + Y = 8$
$-x \quad -x$
$y = \boxed{8 - x}$
$3X + \boxed{Y} = 16$
$3x + 1(8 - x) = 16$
$3x + 8 - x = 16$
$2x + 8 = 16$
$-8 \quad -8$
$\frac{2x}{2} = \frac{8}{2}$
$x = 4$

$x + y = 8$
$4 + y = 8$
$-4 \quad -4$
$y = 4$
$\boxed{(4,4)}$

$3X + 5Y = 10$
$Y - \cancel{X} = 2$
$+\cancel{x} \quad +x$
$y = 2 + x$
$3x + 5(2 + x) = 10$
$3x + 10 + 5x = 10$
$8x + 10 = 10$
$-10 \quad -10$
$\frac{8x}{8} = \frac{0}{8}$
$x = 0$

$y - x = 2$
$y - 0 = 2$
$y = 2$
$\boxed{(0,2)}$

You go to Walgreens and spent $28.00 on Oreos and juice boxes for you and your friends. Each pack of Oreos cost $3.50 and each pack of juice boxes cost $1.50. You purchased 12 items all together. How many packs of oreos and packs of juice boxes did you buy?

x = # of oreo packs
y = # of juice packs

$ \left\{ \begin{array}{l} 3.5x + 1.5y = 28 \\ x + y = 12 \end{array} \right.$ quantity

totals

$-x \quad -x$
$y = 12 - x$

$3.5x + 1.5(12 - x) = 28$
$3.5x + 18 - 1.5x = 28$
$2x + 18 = 28$
$-18 \quad -18$
$\frac{2x}{2} = \frac{10}{2}$
$x = 5$

$x + y = 12$
$5 + y = 12$
$-5 \quad -5$
$y = 7$

$\boxed{\text{5 oreo packs and 7 juice box packs}}$

Solving Systems using Elimination

*Eliminate one of the variables using addition or subtraction, and simplify the rest of the equation doing the same.

-3 and 3 are opposite on the number line,
• opposite so add

$$\begin{array}{r} 4X - 3Y = 1 \\ + \; 2X + 3Y = 17 \\ \hline 6x = 18 \end{array}$$

$\frac{6x}{6} = \frac{18}{6}$

$X = 3$

$4x - 3y = 1$

$4(3) - 3y = 1$

$12 - 3y = 1$

$-12 \quad -12$

$-3y = -11$

$\frac{-3y}{-3} = \frac{-11}{-3}$

$y = \frac{11}{3}$ or $3.\overline{6}$

$\boxed{(3, \frac{11}{3}) \text{ OR } (3, 3.\overline{6})}$

Both positive 4
• same so subtract

$$\begin{array}{r} 3X + 4Y = 7 \\ - \; 2X + 4Y = 4 \\ \hline 1X = 3 \end{array}$$

$X = 3$

$3x + 4y = 7$

$3(3) + 4y = 7$

$9 + 4y = 7$

$-9 \quad -9$

$4y = -2$

$\frac{4y}{4} = \frac{-2}{4}$

$y = -0.5$ or $-\frac{1}{2}$

$\boxed{(3, -0.5) \text{ or } (3, -\frac{1}{2})}$

The admission fee at a small fair is \$1.50 for children and \$4.00 for adults. One day, 2,200 people entered the fair and \$5,050 is collected. How many children and how many adults entered the fair on this day?

total

total

x = # of child tickets
y = # of adult tickets

\$ $\begin{cases} 1.5x + 4y = 5{,}050 \\ x + y = 2{,}200 \end{cases}$

quantity (how many people)

$-x \quad -x$

totals

$y = 2{,}200 - x$

$1.5x + 4(2{,}200 - x) = 5{,}050$

$1.5x + 8{,}800 - 4x = 5{,}050$

$-2.5x + 8{,}800 = 5050$

$-8{,}800 \quad -8{,}800$

$-2.5x = -3750$

$\frac{-2.5x}{-2.5} = \frac{-3750}{-2.5}$

$X = 1{,}500$

$X + y = 2{,}200$

$1{,}500 + y = 2{,}200$

$-1{,}500 \quad -1{,}500$

$y = 700$

$\boxed{\text{1,500 child tickets and 700 adult tickets}}$

*Solve for the variable

4) $-4X + 3 = 5X - 6$

$+4x \quad +4x$

$3 = 9x - 6$

$+6 \quad +6$

$9 = 9x$

$\frac{9}{9} = \frac{9x}{9}$

$1 = X$

5) $3(X - 2) = 7X - 14$

$3x - 6 = 7x - 14$

$-3x \quad -3x$

$-6 = 4x - 14$

$+14 \quad +14$

$\frac{8}{4} = \frac{4x}{4}$ → $X = 2$

6) $-3X + 5 = -3X - 2$

$+3x \quad +3x$

$5 \neq -2$ $\boxed{\text{No solution}}$

7) Solve with Substitution

$X + 3Y = 12$

$-3y \quad -3y$

$X = 12 - 3y$

$3X - 2Y = -19$

$3(12 - 3y) - 2y = -19$

$36 - 9y - 2y = -19$

$36 - 11y = -19$

$-36 \quad -36$

$\frac{-11y}{-11} = \frac{-55}{-11}$

$y = 5$

$x + 3y = 12$

$x + 3(5) = 12$

$x + 15 = 12$

$-15 \quad -15$

$x = -3$

$\boxed{(-3, 5)}$

8) Solve with Elimination

opposite so add

$2X + Y = 15$

$+ \quad X - Y = 45$

$\frac{3x}{3} = \frac{60}{3}$

$x = 20$

$2x + y = 15$

$2(20) + y = 15$

$40 + y = 15$

$-40 \quad -40$

$y = -25$

$\boxed{(20, -25)}$

Word Problems

total for 1st Day

1a) The school that Stefan goes to is selling tickets to a choral performance. On the first day of ticket sales the school sold 3 senior citizen tickets and 1 child ticket for a total of \$38. The school took in \$52 on the second day by selling 3 senior citizen tickets and 2 child tickets. Find the price of a senior citizen ticket and the price of a child ticket.

total for 2nd day

x = price of senior ticket
y = price of child ticket

← same so subtract (can use elimination)

First Day $3x + 1y = 38$ ← totals
2nd day $3x + 2y = 52$ ←

$-1y = -14$

$\frac{-1y}{-1} = \frac{-14}{-1}$

$y = 14$

$3x + 1y = 38$

$3x + 14 = 38$

$\quad -14 \quad -14$

$\frac{3x}{3} = \frac{24}{3}$ $\quad x = 8$

x → Senior tickets cost 8 dollars
y → and child tickets cost 14 dollars

1b) The senior classes at High School A and High School B planned separate trips to New York City. The senior class at High School A rented and filled 1 van and 6 buses with 372 students. High School B rented and filled 4 vans and 12 buses with 780 students. Each van and each bus carried the same number of students. How many students can a van carry? How many students can a bus carry?

total / total

→ x = # of students per van
y = # of students per bus

← totals

A → $1x + 6y = 372$
B → $4\boxed{x} + 12y = 780$

$1x + 6y = 372$

$\quad -6y \quad -6y$

$x = \boxed{372 - 6y}$

(I used substitution) ↳

$4(372 - 6y) + 12y = 780$

$1{,}488 - 24y + 12y = 780$

$1488 - 12y = 780$

$-1488 \quad -1488$

→ $\frac{-12y}{-12} = \frac{-708}{-12}$

$y = 59$

$x + 6y = 372$

$x + 6(59) = 372$

$x + 354 = 372$

$\quad -354 \quad -354$

$x = 18$

18 students per van and 59 students per bus

4) ~~The school that Stefan goes to is selling tickets to a choral performance. On the first day of ticket sales the school sold 3 senior citizen tickets and 1 child ticket for a total of \$38. The school took in \$52 on the second day by selling 3 senior citizen tickets and 2 child tickets. Find the price of a senior citizen ticket and the price of a child ticket.~~

Same as above

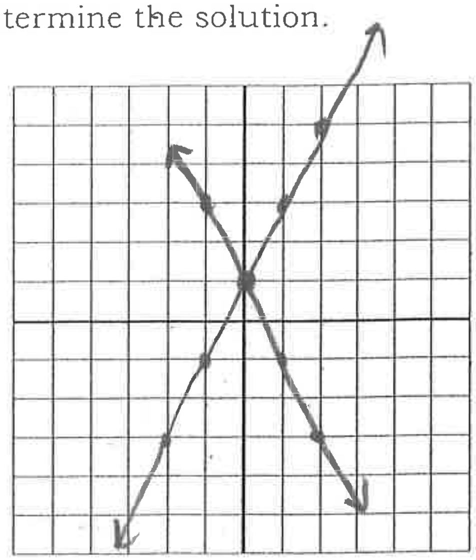
:)

Graph each system of linear equations below to determine the solution.

(1) $y = 2x + 1$
$y = -2x + 1$

$\frac{2}{1}$ ↑ →

$\frac{-2}{1}$ ↓ →



Solution: (0, 1)
x y

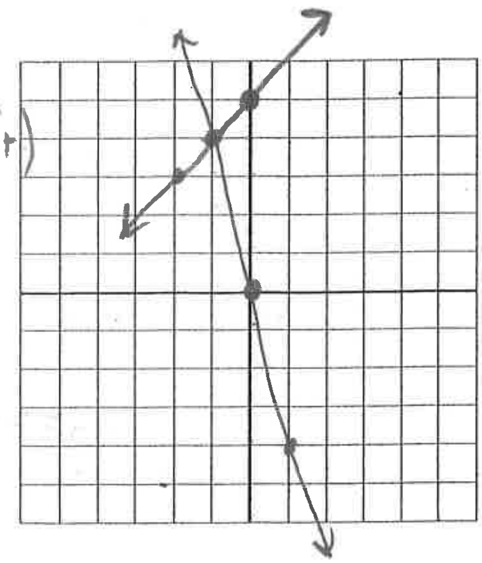
Check your answer:

$y = 2x+1$	$y = -2x+1$
$1 = 2(0)+1$	$1 = -2(0)+1$
$1 = 1$ ✓	$1 = 1$ ✓

(2) $y = -4x + 0$
$y = 1x + 5$

$\frac{-4}{1}$ ↓ → (down and right is same as up & left)

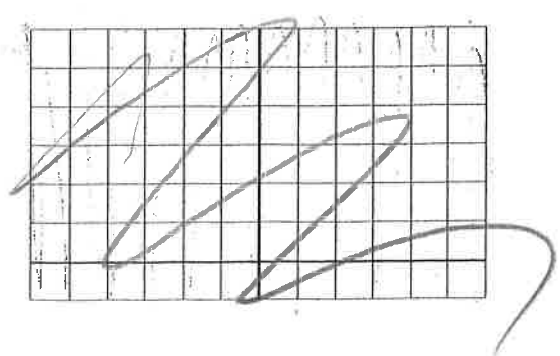
$\frac{1}{1}$



Solution: (-1, 4)
x y

Check your answer:

$y = -4x$	$y = x+5$
$4 = -4(-1)$	$4 = -1+5$
$4 = 4$ ✓	$4 = 4$ ✓



(3) $2y = 6x$

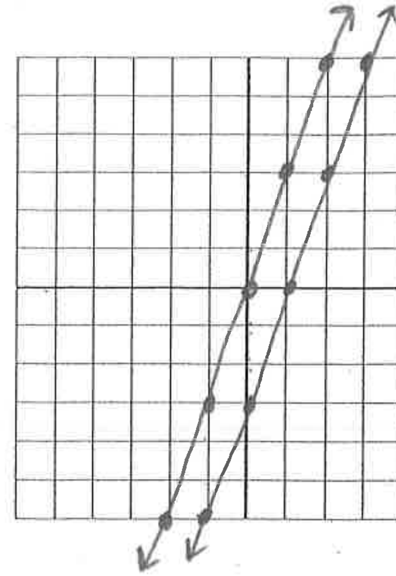
$\frac{2y}{2} = \frac{6x}{2}$

$y = 3x - 3$

$y = \frac{3}{1}x + 0$

$\frac{3\uparrow}{1\rightarrow}$

$\frac{3\uparrow}{1\rightarrow}$



Solution: $\boxed{\text{No solution}}$

(4) $-x + y = 1$

$2x + y = -2$

$+x \quad +x$

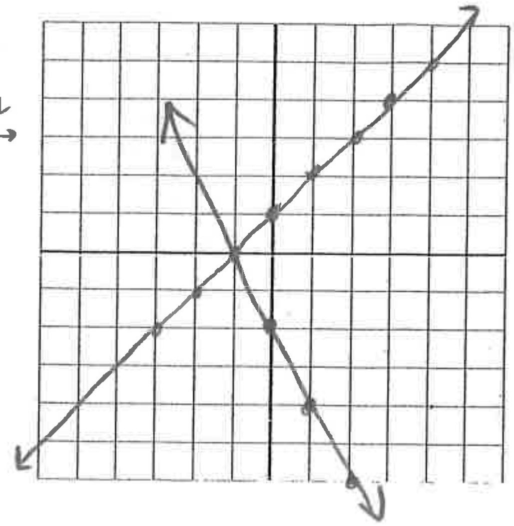
$-2x \quad -2x$

$y = \frac{1}{1}x + 1$

$y = \frac{-2}{1}x - 2$

$\frac{1\uparrow}{1\rightarrow}$

$\frac{-2\downarrow}{1\rightarrow}$



Solution: (-1, 0)

(5) $2x + y = 4$

$-4x - 2y = 6$

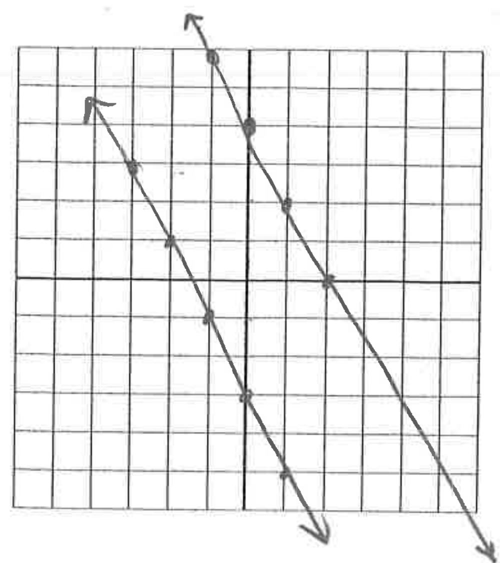
$-2x \quad -2x$

$+4x \quad +4x$

$y = \frac{-2}{1}x + 4$

$\frac{-2y}{-2} = \frac{4x}{-2} + \frac{6}{-2}$

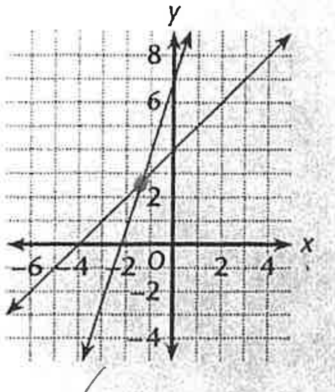
$y = \frac{-2}{1}x - 3$



Solution: $\boxed{\text{No solution}}$

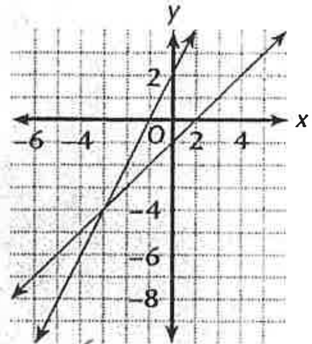
For each system of linear equations graphed below, determine the solution. Estimate where necessary.

(1)



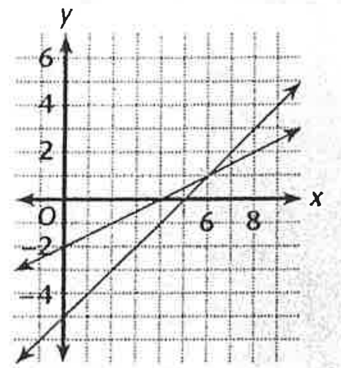
(-1.5, 2.5)

(2)



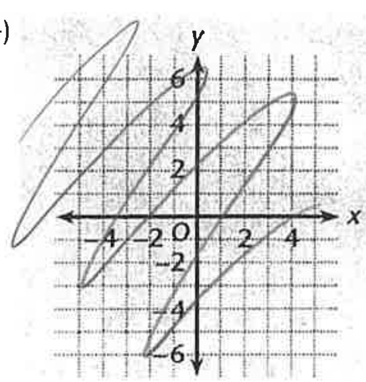
(-3, -4)

(3)

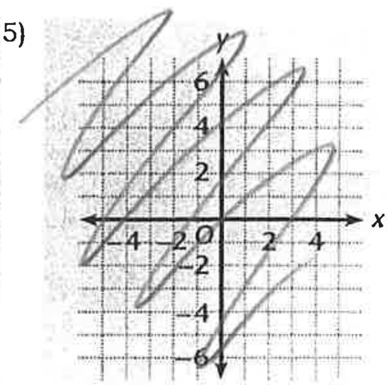


(6, 1)

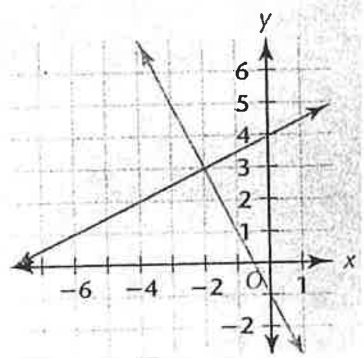
(4)



5)

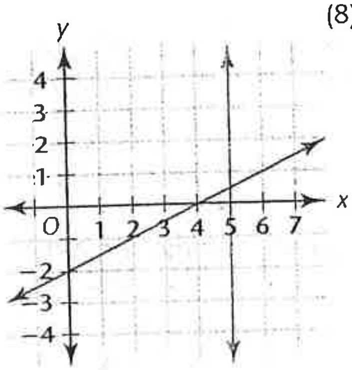


(6)



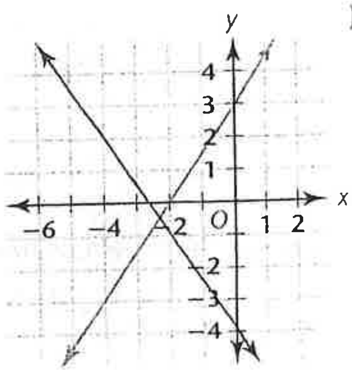
(-2, 3)

(7)



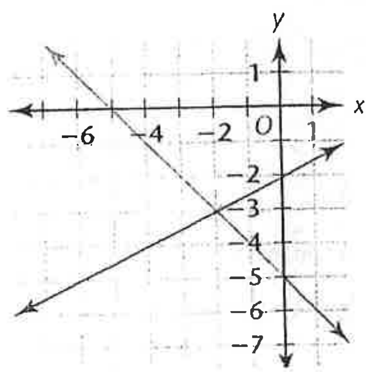
(5, 0.5)

(8)



(-2.5, -0.5)

)



(-2, -3)

Solve the following problems by adding or subtracting to eliminate one of the variables.

1.

opposite add

$$\begin{aligned} 2x - y &= 1 \\ + \quad 4x + y &= 11 \\ \hline 6x &= 12 \end{aligned}$$

$\frac{6x}{6} = \frac{12}{6}$

$x=2$

$2(2) - y = 1$

$4 - y = 1$

$-4 \quad -4$

$\frac{-y}{-1} = \frac{-3}{-1}$

$y = 3$

$\boxed{(2,3)}$

2.

same subtract

$$\begin{aligned} \cancel{4x} + 7y &= 26 \\ - \quad \cancel{4x} - 5y &= 2 \\ \hline 12y &= 24 \end{aligned}$$

$\frac{12y}{12} = \frac{24}{12}$

$y=2$

$4x + 7(2) = 26$

$4x + \cancel{14} = 26$

$-14 \quad -14$

$\frac{4x}{4} = \frac{12}{4}$

$x = 3$

$\boxed{(3,2)}$

3.

same, subtract

$$\begin{aligned} \cancel{x} + 4y &= 11 \\ - \quad \cancel{x} - 6y &= 11 \\ \hline 10y &= 0 \end{aligned}$$

$\frac{10y}{10} = \frac{0}{10}$

$y=0$

$x + 4(0) = 11$

$x + 0 = 11$

$x = 11$

$\boxed{(11,0)}$

4.

Option 1: opposite so add

Option 2: or you can subtract and cancel out the 3y's

$$\begin{aligned} \cancel{-x} + 3y &= 6 \\ + \quad \cancel{x} + 3y &= 18 \\ \hline 6y &= 24 \end{aligned}$$

$\frac{6y}{6} = \frac{24}{6}$

$y = 4$

$-x + 3(4) = 6$

$-x + 12 = 6$

$-12 \quad -12$

$\rightarrow \frac{-x}{-1} = \frac{-6}{-1}$

$x = 6$

$\boxed{(6,4)}$

Summarize: How do you know whether you should add or subtract when using the elimination method?

- 1st make sure all terms are lined up, so x's, y's and = sign are lined up
- Then if you have opposite *(1 pos and 1 neg) coefficients on either the x or on the y you will add.
- If you have same coefficients you will subtract.

Use substitution to solve:

A) $\boxed{y} = \boxed{3x - 13}$
$4x + 5\boxed{y} = 11$

$4x+5(3x-13)=11$

$4x+15x-65=11$

$19x-65=11$
$+65 \quad +65$

$\frac{19x}{19}=\frac{76}{19}$

$x=4$

$y=3(4)-13$
$y=12-13$
$y=-1$

$\boxed{(4,-1)}$

B) $x - 3y = -9$ $\quad \boxed{x} = \boxed{3y-9}$
$+3y \quad +3y$
$5x - 2y = 7$

$5(3y-9)-2y=7$

$15y-45-2y=7$

$13y-45=7$
$+45 \quad +45$

$\frac{13y}{13}=\frac{52}{13}$
$y=4$

$x-3(4)=-9$
$x-12=-9$
$+12 \quad +12$
$x=3$

$\boxed{(3,4)}$

C) $6x - 2\boxed{y} = -4$
$\boxed{y} = \boxed{3x + 2}$

$6x-2(3x+2)=-4$

$6x-6x-4=-4$

$-4=-4$ True

$\boxed{\text{Many Solutions}}$

D) $2x - \boxed{y} = 8$
$\boxed{y} = \boxed{2x - 3}$

$2x-(2x-3)=8$

$2x-2x+3=8$

$3=8$ Not True

$\boxed{\text{No solutions}}$

How can you tell ~~from the substitution method~~ if a system of equation has one solution, no solutions, or infinite solutions?

If a system has one solution, then ... Slopes will be different in slope-intercept form. Or when solving you get an $x=\#$ or $y=\#$

If a system has no solution, then ... Slopes same, y-int different when in $y=mx+b$. Or when solving you get an untrue statement like $3=5$.

If a system has infinite solutions, then...

Slopes are same and y-int are same in $y=mx+b$. Or when solving you get a true statement like $3=3$.

For her parents' anniversary party, Ella is considering using one of two venues. A hotel in ~~Belleville~~ will cost $300 for a reservation, plus $12 per person. A restaurant in the same city will cost $14 per person, in addition to $200 for the reservation. In order to make the best decision, Ella figures out how many attendees it would take to have the venues cost the same amount. What would the total cost be?

x = # of attendees (people)

y = total cost

hotel → $y = 12x + 300$

restaurant → $y = 14x + 200$

total cost is unknown

$$
\begin{aligned}
\cancel{12x} + 300 &= 14x + 200 \\
-\cancel{12x} \quad & \quad -12x \\
300 &= 2x + 200 \\
-200 \quad & \quad -200 \\
\frac{100}{2} &= \frac{2x}{2} \\
x &= 50
\end{aligned}
$$

$y = 12(50) + 300$

$y = 900$

They would cost the same, 900 dollars if 50 people attend.

Mrs. Walton is researching what it would cost to order flower arrangements for a fancy party. She wants one large centerpiece for the head table, and smaller arrangements for the smaller tables. Oakdale Florist charges $15 for each smaller arrangement, plus $50 for the large centerpiece. Fred's Flowers, in contrast, charges $40 for the large centerpiece and $20 per arrangement for the rest. If Mrs. Walton orders a certain number of small arrangements, the cost will be the same at either flower shop. How many small arrangements would that be?

↳ I don't know total cost

x = # of small arrangements

y = total cost

Oakdale → $y = 15x + 50$

Fred's → $y = 20x + 40$

$$
\begin{aligned}
\cancel{15x} + 50 &= 20x + 40 \\
-\cancel{15x} \quad & \quad -15x \\
50 &= 5x + 40 \\
-40 \quad & \quad -40 \\
\frac{10}{5} &= \frac{5x}{5} \\
2 &= x
\end{aligned}
$$

$y = 15(2) + 50$

$y = 30 + 50$

$y = 80$

The cost would be the same, 80 dollars, if 2 small arrangements were ordered.

Use substitution to solve:

A) $y = 3x - 13$
$4x + 5y = 11$

B) $x - 3y = -9$
$5x - 2y = 7$

C) $6x - 2y = -4$
$y = 3x + 2$

D) $2x - y = 8$
$y = 2x - 3$

How can you tell [redacted] if a system of equation has one solution, no solutions, or infinite solutions?

If a system has one solution, then ...

If a system has no solution, then ...

If a system has infinite solutions, then...