Unit Overview
In this unit you will study a variety of ways to solve quadratic functions and systems of equations and apply your learning to analyzing real world problems.

Key Terms
As you study this unit, add these and other terms to your math notebook. Include in your notes your prior knowledge of each word, as well as your experiences in using the word in different mathematical examples. If needed, ask for help in pronouncing new words and add information on pronunciation to your math notebook. It is important that you learn new terms and use them correctly in your class discussions and in your problem solutions.

Math Terms
- quadratic function
- standard form of a quadratic function
- parabola
- vertex of a parabola
- maximum
- minimum
- parent function
- axis of symmetry
- translation
- vertical stretch
- vertical shrink
- transformation
- reflection
- factored form
- zeros of a function
- roots
- completing the square
- discriminant
- imaginary numbers
- imaginary unit
- complex numbers
- piecewise-defined function
- nonlinear system of equations

Embedded Assessments
This unit has three embedded assessments, following Activities 30, 33, and 35. They will allow you to demonstrate your understanding of graphing, identifying, and modeling quadratic functions, solving quadratic equations and solving nonlinear systems of equations.

Embedded Assessment 1:
Graphing Quadratic Functions p. 453

Embedded Assessment 2:
Solving Quadratic Equations p. 493

Embedded Assessment 3:
Solving Systems of Equations p. 519
Write your answers on notebook paper. Show your work.

1. Determine each product.
   a. \((x - 2)(3x + 5)\)
   b. \(2y(y + 6)(y - 1)\)

2. Factor each polynomial.
   a. \(2x^2 + 14x\)
   b. \(3x^2 - 75\)
   c. \(x^2 + 7x + 10\)

3. If \(f(x) = 3x - 5\), find each value.
   a. \(f(4)\)
   b. \(f(-2)\)

4. Solve the equation. \(4x - 5 = 19\)

5. Solve the inequality.
   \(\frac{1}{3}x + 9 > 13\)

6. Explain how to graph \(2x + y = 4\).

The following graph compares calories burned when running and walking at constant rates of 10 mi/h and 2 mi/h, respectively.

7. What does the ordered pair \((1.5, 300)\) represent on this graph?

8. How many calories would be burned after four hours when running and after four hours when walking?
Learning Targets:
- Model a real-world situation with a quadratic function.
- Identify quadratic functions.
- Write a quadratic function in standard form.

SUGGESTED LEARNING STRATEGIES: Create Representations, Interactive Word Wall, Marking the Text, Look for a Pattern, Discussion Groups

Coach Wentworth coaches girls’ soccer and teaches algebra. Soccer season is starting, and she needs to mark the field by chalking the touchlines and goal lines for the soccer field. Coach Wentworth can mark 320 yards for the total length of all the touchlines and goal lines combined. She would like to mark the field with the largest possible area.

FIFA regulations require that all soccer fields be rectangular in shape.

1. How is the perimeter of a rectangle determined? How is the area of a rectangle determined?
2. Complete the table below for rectangles with the given side lengths. The first row has been completed for you.

<table>
<thead>
<tr>
<th>Length (yards)</th>
<th>Width (yards)</th>
<th>Perimeter (yards)</th>
<th>Area (square yards)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>150</td>
<td>320</td>
<td>1500</td>
</tr>
<tr>
<td>20</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>40</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>60</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>80</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>100</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>120</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>140</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>150</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>l</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3. **Express regularity in repeated reasoning.** Describe any patterns you observe in the table above.

4. Is a 70-yd by 90-yd rectangle the same as a 90-yd by 70-yd rectangle? Explain your reasoning.

5. Graph the data from the table in Item 2 as ordered pairs.
6. Use the table and the graph to explain why the data in Items 2 and 5 are not linear.

7. Describe any patterns you see in the graph above.

8. What appears to be the largest area from the data in Items 2 and 5?

9. Write a function $A(l)$ that represents the area of a rectangle whose length is $l$ and whose perimeter is 320.

The function $A(l)$ is called a **quadratic function** because the greatest degree of any term is 2 (an $x^2$ term). The **standard form of a quadratic function** is $y = ax^2 + bx + c$ or $f(x) = ax^2 + bx + c$, where $a$, $b$, and $c$ are real numbers and $a \neq 0$.

10. Write the function $A(l)$ in standard form. What are the values of $a$, $b$, and $c$?
Check Your Understanding

11. For the function \( f(x) = x^2 + 2x + 3 \), create a table of values for \( x = -3, -2, -1, 0, 1 \). Then sketch a graph of the quadratic function on grid paper.

12. Barry needs to find the area of a rectangular room with a width that is 2 feet longer than the length. Write an expression for the area of the rectangle in terms of the length.

13. Critique the reasoning of others. Sally states that the equation \( g(x) = x^3 + 10x^2 − 3x \) represents a quadratic function. Explain why Sally is incorrect.

14. Write the quadratic function \( f(x) = (3 - x)^2 \) in standard form.

LESSON 29-1 PRACTICE

15. Create tables to graph \( y = 3x \) and \( y = 3x^2 \) on grid paper. Explain the differences between the graphs.

16. Pierre uses the function \( r(t) = t + 2 \) to model his rate \( r \) in mi/h \( t \) minutes after leaving school. Complete the table and use the data points to graph the function.

<table>
<thead>
<tr>
<th>( t )</th>
<th>( r(t) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

17. Pierre uses the function \( d(t) = t(t + 2) \) to model his distance \( d \) in miles from home. Add another column to your table in Item 16 to represent \( d(t) \). Sketch a graph of \( d(t) \) on the same coordinate plane as the graph of \( r(t) \).

18. What types of functions are represented in Items 16 and 17?

19. Write each quadratic function in standard form.
   a. \( g(x) = x(x - 2) + 4 \)
   b. \( f(t) = 3 - 2t^2 + t \)

20. Attend to precision. Determine whether each function is a quadratic function. Justify your responses.
   a. \( S(r) = 2\pi r^2 + 20\pi r \)
   b. \( f(a) = \frac{a^2 + 4a - 3}{2} \)
   c. \( f(x) = 4x^2 - 3x + 2 \)
   d. \( g(x) = 3x^2 + 2x - 1 \)
   e. \( f(x) = 4x^2 - 3 \)
   f. \( h(x) = \frac{4}{x^2} - 3x + 2 \)
Lesson 29-2
Graphing and Analyzing a Quadratic Function

Learning Targets:
• Graph a quadratic function.
• Interpret key features of the graph of a quadratic function.

SUGGESTED LEARNING STRATEGIES: Interactive Word Wall, Create Representations, Construct an Argument, Marking the Text, Discussion Groups

1. Use a graphing calculator to graph $A(l)$ from Item 9 in Lesson 29-1. Sketch the graph on the grid below.

![Graph of A(l)](image)

The graph of a quadratic function is a curve called a **parabola**. A parabola has a point at which a maximum or minimum value of the function occurs. That point is called the **vertex of a parabola**. The $y$-value of the vertex is the **maximum** or **minimum** of the function.

2. Identify the vertex of the graph of $A(l)$ in Item 1. Does the vertex represent a maximum or a minimum of the function?

3. Examine the graph of $A(l)$. For what values of the length is the area increasing? For what values of the length is the area decreasing?

**TECHNOLOGY TIP**
Be sure that the RANGE on your calculator’s graph matches the range shown in the grid.

**CONNECT TO AP**
AP Calculus students find these same maximum or minimum values of functions in optimization problems.
4. Describe the point where the area changes from increasing to decreasing.

5. Use the table, the graph, and/or the function to determine the reasonable domain and range of the function $A(l)$. Describe each using words and an inequality.

FIFA regulations state that the length of the touchline of a soccer field must be greater than the length of the goal line.

6. **Reason abstractly.** Can Coach Wentworth use the rectangle that represents the largest area of $A(l)$ for her soccer field? Explain why or why not.
FIFA regulations also state that the length of the touchlines of a soccer field must be at least 100 yds, but no more than 130 yds. The goal lines must be at least 50 yds, but no more than 100 yds.

7. **Construct viable arguments.** Determine the dimensions of the FIFA regulation soccer field with the largest area and a 320-yd perimeter. Support your reasoning with multiple representations.

8. Consider the quadratic function \( f(x) = x^2 - 2x - 3 \).
   a. Write the function in factored form by factoring the polynomial \( x^2 - 2x - 3 \).
   
   b. To find the \( x \)-intercepts of \( f(x) \), use the factored form of \( f(x) \) and solve the equation \( f(x) = 0 \).
   
   c. A parabola is symmetric over the vertical line that contains the vertex. How do you think the \( x \)-coordinate of the vertex relates to the \( x \)-coordinates of the \( x \)-intercepts? Use the symmetry of a parabola to support your answer.
   
   d. Write the vertex of the quadratic function.
Lesson 29-2
Graphing and Analyzing a Quadratic Function

Check Your Understanding

9. Complete the table for the quadratic function \( f(x) = -x^2 - 4x - 3 \). Then graph the function.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>5</td>
</tr>
<tr>
<td>-3</td>
<td>0</td>
</tr>
<tr>
<td>-2</td>
<td>-3</td>
</tr>
<tr>
<td>-1</td>
<td>-4</td>
</tr>
<tr>
<td>0</td>
<td>-3</td>
</tr>
</tbody>
</table>

10. Identify the maximum or minimum value of the quadratic function in Item 9.

11. Consider the quadratic function \( y = -x^2 + 2x - 3 \).
   a. Create a table of values for the function for domain values 0, 1, 2, 3, and 4.
   b. Sketch a graph of the function. Identify and label the vertex. What is the maximum value of the function?
   c. Use inequalities to write the domain, range, and values of \( x \) for which \( y \) is decreasing.

LESSON 29-2 PRACTICE

12. Write the quadratic function \( g(x) = x(x - 2) + 4 \) in standard form.

For Items 13–16, use the quadratic function \( f(x) = x^2 + 6x + 5 \).

13. Create a table of values and graph \( f(x) \).

14. Use your graph to identify the maximum or minimum value of \( f(x) \).

15. Write the domain and range of \( f(x) \) using inequalities.

16. Determine the values of \( x \) for which \( f(x) \) is increasing.

17. Make use of structure. Sketch a graph of a quadratic function with a maximum. Now sketch another graph of a quadratic function with a minimum. Explain the difference between the increasing and decreasing behavior of the two functions.
ACTIVITY 29 PRACTICE
Write your answers on notebook paper.
Show your work.

Lesson 29-1

1. The base of a rectangular window frame must be 1 foot longer than the height. Which of the following is an equation for the area of the window in terms of the height?
   A. \( A(h) = h + 1 \)
   B. \( A(h) = (h + 1)h \)
   C. \( A(h) = h^2 - 1 \)
   D. \( A(h) = h^2 + h + 1 \)

2. Which of the following is an equation for the area of an isosceles right triangle, in terms of the base?
   A. \( A(b) = b^2 \)
   B. \( A(b) = \frac{1}{2} b^2 \)
   C. \( A(b) = \frac{\sqrt{b^2}}{2} \)
   D. \( A(b) = \frac{1}{2} b \)

3. Ben is creating a triangle that has a base that is twice the length of the height.
   a. Write an expression for the base of the triangle in terms of the height.
   b. Write a function for the area, \( A(h) \), of the triangle in terms of the height.
   c. Complete the table and then graph the function.

<table>
<thead>
<tr>
<th>( h )</th>
<th>( A(h) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

Use the following information for Items 4–8.
Jenna is in charge of designing the screen for a new smart phone. The design specs call for a rectangular screen that has an outside perimeter of 12 inches.

4. Complete the table for the screen measurements.

<table>
<thead>
<tr>
<th>Width, ( w )</th>
<th>Length, ( l )</th>
<th>Area, ( A(w) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5. Write a function \( A(w) \) for the area of the screen in terms of the width.

6. Graph the function \( A(w) \). Label each axis.

7. Determine the domain and range of the function.

8. Determine the maximum area of the screen that Jenna can design. What are the dimensions of this screen?

9. Samantha’s teacher writes the function \( f(x) = 2x^3 - 2x(3 - x + x^2) + 3 \).
   a. Barry tells Samantha that the function cannot be quadratic because it contains the term \( 2x^3 \). What should Barry do to the function before making this assumption?
   b. Is the function a quadratic function? Explain.
10. Identify whether each function is quadratic.
   a. $y = 2x - 3^2$
   b. $y = 3x^2 - 2x$
   c. $y = 2 - \frac{3}{x^2} + x$

11. State whether the data in each table are linear. Explain why or why not.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>5</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>-1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>-4</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>-7</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

12. Write each quadratic function in standard form.
   a. $y = 3x - 5 + x^2$
   b. $y = 6 - 5x^2$
   c. $y = -0.5x + \frac{3}{4}x^2 - \pi$

13. For each function, complete the table of values. Graph the function and identify the maximum or minimum of the function.
   a. $y = x^2 + 4x - 1$

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td></td>
</tr>
<tr>
<td>-2</td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

b. $y = -x^2 + 8x - 13$

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
</tbody>
</table>

14. Use your graphing calculator to graph several functions $y = ax^2 + b$ using positive and negative values of $a$ and $b$. Do the signs of $a$ and $b$ appear to affect whether the function has a minimum or maximum value? Make a conjecture.

15. As part of her math homework, Kylie is graphing a quadratic function. After plotting several points, she notices that the dependent values are increasing as the independent values increase. She reasons that the function will eventually reach a maximum value and then begin to decrease. Is Kylie correct? Why or why not?
Graphing Quadratic Functions
Transformers
Lesson 30-1 Translations of the Quadratic Parent Function

Learning Targets:
• Graph translations of the quadratic parent function.
• Identify and distinguish among transformations.

SUGGESTED LEARNING STRATEGIES: Look for a Pattern, Create Representations, Sharing and Responding, Quickwrite, Think-Pair-Share

The function \( y = x^2 \) or \( f(x) = x^2 \) is the quadratic parent function.

1. Complete the table for \( y = x^2 \). Then graph the function.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td></td>
</tr>
<tr>
<td>-2</td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>

2. Use words such as vertex, maximum, minimum, increasing, and decreasing to describe the graph of \( y = x^2 \).

3. The line that passes through the vertex of a parabola and divides the parabola into two symmetrical parts is the parabola’s axis of symmetry.
   a. Draw the axis of symmetry for the graph of \( y = x^2 \) as a dashed line on the graph in Item 1.
   b. Write the equation for the axis of symmetry you drew in Item 1.

**MATH TERMS**
A parent function is the most basic function of a particular category or type. For example, the linear parent function is \( y = x \) or \( f(x) = x \).
4. Complete the second and third columns of the table below. (Leave the fourth column blank for now.) Use your results to explain why the function $y = x^2$ is not linear.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y = x^2$</th>
<th>Difference Between Consecutive $y$-Values (“First Differences”)</th>
<th>“Second Differences”</th>
</tr>
</thead>
<tbody>
<tr>
<td>−3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>−2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>−1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5. The fourth column in the table in Item 4 is titled “Second Differences.” Complete the fourth column by finding the change in consecutive values in the third column. What do you notice about the values?

6. a. Complete the table for $f(x) = x^2$ and $g(x) = x^2 + 3$. Then graph each function on the same coordinate grid.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x) = x^2$</th>
<th>$g(x) = x^2 + 3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>−3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>−2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>−1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b. Identify the vertex, domain, range, and axis of symmetry for each function.

<table>
<thead>
<tr>
<th>$f(x) = x^2$</th>
<th>$g(x) = x^2 + 3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertex</td>
<td></td>
</tr>
<tr>
<td>Domain</td>
<td></td>
</tr>
<tr>
<td>Range</td>
<td></td>
</tr>
<tr>
<td>Axis of symmetry</td>
<td></td>
</tr>
</tbody>
</table>
Lesson 30-1
Translations of the Quadratic Parent Function

7. a. Complete the table for \( f(x) = x^2 \) and \( h(x) = x^2 - 4 \). Then graph each function on the same coordinate grid.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) = x^2 )</th>
<th>( h(x) = x^2 - 4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>9</td>
<td>5</td>
</tr>
<tr>
<td>-2</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>-1</td>
<td>1</td>
<td>-3</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>-4</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>-3</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
<td>5</td>
</tr>
</tbody>
</table>

b. Identify the vertex, domain, range, and axis of symmetry for each function.

<table>
<thead>
<tr>
<th>( f(x) = x^2 )</th>
<th>( h(x) = x^2 - 4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertex</td>
<td></td>
</tr>
<tr>
<td>Domain</td>
<td></td>
</tr>
<tr>
<td>Range</td>
<td></td>
</tr>
<tr>
<td>Axis of symmetry</td>
<td></td>
</tr>
</tbody>
</table>

8. Compare the functions \( g(x) \) and \( h(x) \) in Items 6 and 7 to the parent function \( f(x) = x^2 \). Describe any patterns you notice in the following.

a. equations

b. tables

c. graphs
d. vertices

e. domain and range

f. axes of symmetry

The changes to the parent function in Items 6 and 7 are examples of vertical translations. A translation of a graph is a change that shifts the graph horizontally, vertically, or both. A translation does not change the shape of the graph.

9. Make use of structure. How does the value of \( k \) in the equation \( g(x) = x^2 + k \) change the graph of the parent function \( f(x) = x^2 \)?

Check Your Understanding

For Items 10 and 11, predict the translations of the graph of \( f(x) = x^2 \) for each function. Confirm your predictions by graphing each equation on the same coordinate grid as the graph of the parent function.

10. \( g(x) = x^2 - 2 \)  
11. \( h(x) = x^2 + 4 \)

12. The graphs of two functions are shown below, along with the graph of the parent function. Determine the value of \( c \) for each function.

a. \( g(x) = x^2 + c \)  
b. \( h(x) = x^2 + c \)
Lesson 30-1
Translations of the Quadratic Parent Function

13. a. Complete the table for \( f(x) = x^2 \) and \( k(x) = (x + 3)^2 \). Then graph both functions on the same coordinate grid.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) = x^2 )</th>
<th>( k(x) = (x + 3)^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
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<td></td>
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<tr>
<td>1</td>
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<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b. Identify the vertex, domain, range, and axis of symmetry for each function.

<table>
<thead>
<tr>
<th>( f(x) = x^2 )</th>
<th>( k(x) = (x + 3)^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertex</td>
<td></td>
</tr>
<tr>
<td>Domain</td>
<td></td>
</tr>
<tr>
<td>Range</td>
<td></td>
</tr>
<tr>
<td>Axis of symmetry</td>
<td></td>
</tr>
</tbody>
</table>

14. a. Complete the table for \( f(x) = x^2 \) and \( p(x) = (x - 4)^2 \). Then graph both functions on the same coordinate grid.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) = x^2 )</th>
<th>( p(x) = (x - 4)^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
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<td></td>
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<td>2</td>
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<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
b. Identify the vertex, domain, range, and axis of symmetry for each function.

<table>
<thead>
<tr>
<th></th>
<th>( f(x) = x^2 )</th>
<th>( p(x) = (x - 4)^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertex</td>
<td></td>
<td></td>
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<tr>
<td>Domain</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Range</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Axis of symmetry</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

15. Compare the functions \( k(x) \) and \( p(x) \) in Items 13 and 14 to the parent function \( f(x) = x^2 \). Describe any patterns you notice in the following.

a. equations

b. graphs

c. vertices

d. domain and range

e. axes of symmetry
Lesson 30-1
Translations of the Quadratic Parent Function

The changes to the parent function in Items 13 and 14 are examples of horizontal translations.

16. **Express regularity in repeated reasoning.** How does the value of \( h \) in the equation \( k(x) = (x + h)^2 \) change the graph of the parent function \( k(x) = x^2 \)?

---

**Check Your Understanding**

For Items 17 and 18, predict the translations of the graph of \( f(x) = x^2 \) for each equation. Confirm your predictions by graphing each equation on the same coordinate grid as the graph of the parent function.

17. \( k(x) = (x - 2)^2 \)  
18. \( p(x) = (x + 1)^2 \)

19. The graphs of two functions are shown below, along with the graph of the parent function. Determine the value of \( h \) for each function. Then write each function in standard form.

   a. \( k(x) = (x + h)^2 \)  
   b. \( p(x) = (x + h)^2 \)

---

**LESSON 30-1 PRACTICE**

20. The graph of \( g(x) \) is a vertical translation 2 units up from the graph of \( f(x) = x^2 \). Which of the following equations describes \( g(x) \)?
   A. \( g(x) = x^2 + 2 \)  
   B. \( g(x) = x^2 - 2 \)  
   C. \( g(x) = (x + 2)^2 \)  
   D. \( g(x) = (x - 2)^2 \)

21. For each function, use translations to graph the function on the same coordinate grid as the parent function \( f(x) = x^2 \). Then identify the vertex, domain, range, and axis of symmetry.
   a. \( g(x) = x^2 + 1 \)  
   b. \( h(x) = x^2 - 1 \)  
   c. \( k(x) = (x + 1)^2 \)  
   d. \( n(x) = (x - 1)^2 \)

22. **Attend to precision.** Write an ordered pair that represents the vertex of the graph of each quadratic function.
   a. \( f(x) = x^2 + c \)  
   b. \( f(x) = (x - k)^2 \)
Lesson 30-2
Stretching and Shrinking the Quadratic Parent Function

Learning Targets:
• Graph vertical stretches and shrinks of the quadratic parent function.
• Identify and distinguish among transformations.

SUGGESTED LEARNING STRATEGIES: Create Representations, Look for a Pattern, Sharing and Responding, Think-Pair-Share, Discussion Groups

1. Complete the table for \( f(x) = x^2 \) and \( g(x) = 2x^2 \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) = x^2 )</th>
<th>( g(x) = 2x^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td></td>
<td></td>
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<tr>
<td>-1</td>
<td></td>
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<tr>
<td>0</td>
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<tr>
<td>1</td>
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<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a. The graph of \( f(x) = x^2 \) is shown below. Graph \( g(x) = 2x^2 \) on the same coordinate grid.

b. Identify the vertex, domain, range, and axis of symmetry for each function.

<table>
<thead>
<tr>
<th>( f(x) = x^2 )</th>
<th>( g(x) = 2x^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertex</td>
<td></td>
</tr>
<tr>
<td>Domain</td>
<td></td>
</tr>
<tr>
<td>Range</td>
<td></td>
</tr>
<tr>
<td>Axis of symmetry</td>
<td></td>
</tr>
</tbody>
</table>
Lesson 30-2
Stretching and Shrinking the Quadratic Parent Function

2. Complete the table for \( f(x) = x^2 \) and \( h(x) = \frac{1}{2} x^2 \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) = x^2 )</th>
<th>( h(x) = \frac{1}{2} x^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-1</td>
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<td></td>
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<tr>
<td>0</td>
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<td>2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a. The graph of \( f(x) = x^2 \) is shown below. Graph \( h(x) = \frac{1}{2} x^2 \) on the same coordinate grid.

b. Identify the vertex, domain, range, and axis of symmetry for each function.

<table>
<thead>
<tr>
<th>( f(x) = x^2 )</th>
<th>( h(x) = \frac{1}{2} x^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertex</td>
<td></td>
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<tr>
<td>Domain</td>
<td></td>
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<tr>
<td>Range</td>
<td></td>
</tr>
<tr>
<td>Axis of symmetry</td>
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</tr>
</tbody>
</table>

3. Compare the functions \( g(x) \) and \( h(x) \) in Items 1 and 2 to the parent function \( f(x) = x^2 \). Describe any patterns you notice in the following.

a. equations
b. tables

c. graphs

d. vertex, domain and range, and axes of symmetry

The change to the parent function in Item 1 is a **vertical stretch** by a factor of 2 and the change in Item 2 is a **vertical shrink** by a factor of $\frac{1}{2}$. A vertical stretch or shrink changes the shape of the graph.

4. **Reason quantitatively.** For $a > 0$, how does the value of $a$ in the equation $g(x) = ax^2$ change the graph of the parent function $f(x) = x^2$?

5. A change in the position, size, or shape of a parent graph is a **transformation.** Identify the transformations that have been introduced so far in this activity.

Vertical stretching and shrinking also applies to linear functions. Below is the graph of the parent linear function $f(x) = x$.

6. Graph the functions $g(x) = 2x$ and $h(x) = \frac{1}{2}x$ on the coordinate grid above. How do these graphs compare with the parent linear function?
Lesson 30-2
Stretching and Shrinking the Quadratic Parent Function

Check Your Understanding

For Items 7 and 8, predict the change from the graph of \( f(x) = x^2 \) for each function. Confirm your predictions by graphing each function on the same coordinate grid as the graph of the parent function.

7. \( g(x) = 3x^2 \)
8. \( h(x) = \frac{1}{4}x^2 \)

Write the equation for each transformation of the graph of \( f(x) = x^2 \).

9. a vertical stretch by a factor of 5
10. a vertical shrink by a factor of \( \frac{1}{5} \)

LESSON 30-2 PRACTICE

11. The graph of \( g(x) \) is a vertical stretch of the graph of \( f(x) = x^2 \) by a factor of 7. Which of the following equations describes \( g(x) \)?
   A. \( g(x) = x^2 + 7 \)
   B. \( g(x) = 7x^2 \)
   C. \( g(x) = (x + 7)^2 \)
   D. \( g(x) = \frac{1}{7}x^2 \)

12. For each function, use transformations to graph the function on the same coordinate grid as the parent function \( f(x) = x^2 \).
   a. \( g(x) = 4x^2 \)
   b. \( h(x) = \frac{1}{4}x^2 \)
   c. \( k(x) = 0.5x^2 \)
   d. \( p(x) = \frac{3}{4}x^2 \)

13. Reason abstractly. Chen begins with a quadratic data set, which contains the vertex \((0, 0)\). He multiplies every value in the range by 3 to create a new data set. Which of the following statements are true?
   A. The function that represents the new data set must be \( y = 3x^2 \).
   B. The graphs of the original data set and the new data set will have the same vertex.
   C. The graph of the new data set will be a vertical stretch of the graph of the original data set.
   D. The graph of the new data set will be translated up 3 units from the graph of the original data set.

14. How are the graphs of linear functions \( f(x) = x \), \( k(x) = \frac{3}{4}x \), and \( t(x) = \frac{4}{3}x \) the same? How are they different?
Learning Targets:
- Graph reflections of the quadratic parent function.
- Identify and distinguish among transformations.
- Compare functions represented in different ways.

SUGGESTED LEARNING STRATEGIES: Look for a Pattern, Create Representations, Predict and Confirm, Sharing and Responding, Think-Pair-Share

1. The quadratic parent function \( f(x) = x^2 \) is graphed below. Graph \( g(x) = -x^2 \) on the same coordinate grid.

2. Compare \( g(x) \) and its graph to the parent function \( f(x) = x^2 \).

The change to the parent function in Item 1 is a reflection over the \( x \)-axis. A reflection of a graph is the mirror image of the graph over a line. A reflection preserves the shape of the graph.

3. How does the sign of \( a \) in the function \( g(x) = ax^2 \) affect the graph?

A graph may represent more than one transformation of the graph of the parent function. The order in which multiple transformations are performed is determined by the order of operations as indicated in the equation.
Lesson 30-3
Multiple Transformations of the Quadratic Parent Function

4. How is the graph of \( f(x) = x^2 \) transformed to produce the graph of \( g(x) = 2x^2 - 5 \)?

5. Use the transformations you described in Item 4 to graph the function \( g(x) = 2x^2 - 5 \).

6. How would you transform the graph of \( f(x) = x^2 \) to produce the graph of the function \( h(x) = -\frac{1}{2}x^2 + 4 \)?

7. Use the transformations you described in Item 6 to graph the function \( h(x) = -\frac{1}{2}x^2 + 4 \). Identify the vertex and axis of symmetry of the parabola.
8. How would you transform the graph of \( f(x) = x^2 \) to produce the graph of the function \( k(x) = -\frac{1}{2} (x + 4)^2 \)?

9. Use the transformations you described in Item 8 to graph the function \( k(x) = -\frac{1}{2} (x + 4)^2 \). Identify the vertex and axis of symmetry of the parabola.

10. **Reason abstractly.** The graph of \( g(x) = a(x - h)^2 + k \) represents multiple transformations of the graph of the parent function, \( f(x) = x^2 \). Describe how each value transforms the graph of \( g(x) \) from the graph of \( f(x) \).
   a. \( k \)
   b. \( h \)
   c. \(|a|\)
   d. the sign of \( a \)
Lesson 30-3
Multiple Transformations of the Quadratic Parent Function

11. Examine the function \( r(x) = 2(x - 4)^2 + 3 \).
   a. Describe the transformations from the graph of \( f(x) = x^2 \) to the graph of \( r(x) = 2(x - 4)^2 + 3 \).

   b. Use the transformations you described in Part (a) to graph the function.

   ![Graph of the function](image)

   c. Identify the vertex and axis of symmetry of the parabola.

12. The vertex of the graph of \( f(x) = x^2 \) is \((0, 0)\).
   a. Describe how transformations can be used to determine the vertex of the graph of \( g(x) = a(x - h)^2 + k \) without graphing.

   b. Identify the vertex of the graph of \( y = 3(x - 4)^2 + 7 \).

Check Your Understanding

13. Without graphing, determine the vertex of the graph of each function.
   a. \( y = x^2 + 6 \)
   b. \( y = 3x^2 - 8 \)
   c. \( y = (x - 4)^2 \)
   d. \( y = (x - 2)^2 + 1 \)
   e. \( y = 2(x + 2)^2 - 5 \)
Example A

For the quadratic function shown in the graph, write the equation in standard form.

Step 1: Identify the vertex.
\((h, k) = (-1, 1)\)

Step 2: Choose another point on the graph.
\((x, y) = (0, 3)\)

Step 3: Substitute the known values.
\[ y = a(x - h)^2 + k \]
\[ 3 = a(0 - (-1))^2 + 1 \]

Step 4: Solve for \(a\).
\[ 3 = a(1)^2 + 1 \]
\[ 3 = a + 1 \]
\[ 2 = a \]

Step 5: Write the equation of the function by substituting the values of \(a, h,\) and \(k\).
\[ y = a(x - h)^2 + k \]
\[ y = 2(x + 1)^2 + 1 \]

Step 6: Multiply and combine like terms to write the equation in standard form.
\[ y = 2(x + 1)^2 + 1 \]
\[ y = 2(x^2 + 2x + 1) + 1 \]
\[ y = 2x^2 + 4x + 2 + 1 \]
\[ y = 2x^2 + 4x + 3 \]

Solution: \(y = 2x^2 + 4x + 3\)

Try These A

For each quadratic function, write its equation in standard form.

a. 

b. 
Lesson 30-3
Multiple Transformations of the Quadratic Parent Function

The vertex of a parabola represents the minimum value of the quadratic function if the parabola opens upward. The vertex of a parabola represents the maximum value of the quadratic function if the parabola opens downward.

14. Make sense of problems. The equation and graph below represent two different quadratic functions.

Function 1: \( y = x^2 - 3 \)

Function 2:

Which function has the greater minimum value? Justify your response.

15. The verbal description and the table below represent two different quadratic functions.

Function 1: The graph of function 1 is the graph of the parent function stretched vertically by a factor of 4 and translated 1 unit up.

Function 2:

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-5</td>
<td>4</td>
</tr>
<tr>
<td>-4</td>
<td>1</td>
</tr>
<tr>
<td>-3</td>
<td>0</td>
</tr>
<tr>
<td>-2</td>
<td>1</td>
</tr>
<tr>
<td>-1</td>
<td>4</td>
</tr>
</tbody>
</table>

Which function has the greater minimum value? Justify your response.
Lesson 30-3
Multiple Transformations of the Quadratic Parent Function

Check Your Understanding

16. Which of the following represents a quadratic function whose minimum value is less than the minimum value of the function $y = 3x^2 + 4$?
   A. $y = 2(x + 1)^2 + 6$
   B. The function whose graph is translated right 1 unit and up 4 units from the graph of the parent function
   C. $y = 2x^2 + 3$
   D. $y = 3x^2 + 4$

17. For each function, identify how its graph has been transformed from the graph of the parent function $f(x) = x^2$. Then graph each function.
   a. $g(x) = \frac{1}{2}x^2 + 3$
   b. $h(x) = -x^2 + 4$

18. Write the equation of the function whose graph has been transformed from the graph of the parent function as described.
   a. Translated up 9 units and right 2 units
   b. Stretched vertically by a factor of 5 and translated down 12 units

19. Write an equation for each function.
   a. $y = 2x^2 + 3$
   b. $y = -x^2 + 4$

20. Critique the reasoning of others. Kumara graphs a quadratic function, and the vertex of the graph is $(2, 3)$. Phillip states that his function $y = -x^2 + 7$ has a greater maximum value than Kumara’s. Is Phillip correct? Justify your response.
ACTIVITY 30 PRACTICE
Write your answers on notebook paper. Show your work.

Lesson 30-1
For each quadratic function, describe the transformation of its graph from the graph of the parent function \( f(x) = x^2 \).

1. \( g(x) = x^2 + 7 \)
2. \( y = x^2 - \frac{3}{5} \)
3. \( h(x) = (x + 4)^2 \)
4. \( y = (x - \frac{1}{2})^2 \)

5. The graph of which of the following has a vertex at the point \((0, 8)\)?
   A. \( y = x^2 + 8 \)
   B. \( y = x^2 - 8 \)
   C. \( y = (x + 8)^2 \)
   D. \( y = (x - 8)^2 \)

6. Identify the range of the function \( y = x^2 - 3 \).

7. Determine the equation of the axis of symmetry of the graph of \( y = x^2 - 3 \).

8. Identify the range of the function \( y = (x - 11)^2 \).

9. Determine the equation of the axis of symmetry of the graph of \( y = (x - 11)^2 \).

10. Write the equation of each quadratic function.

   a. 
   
   ![Graph of a quadratic function]

   b. The quadratic function whose graph is translated 10 units to the left from the graph of the parent function \( f(x) = x^2 \)

   c. 
   
<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>4.5</td>
</tr>
<tr>
<td>-1</td>
<td>1.5</td>
</tr>
<tr>
<td>0</td>
<td>0.5</td>
</tr>
<tr>
<td>1</td>
<td>1.5</td>
</tr>
<tr>
<td>2</td>
<td>4.5</td>
</tr>
</tbody>
</table>
Lesson 30-2

11. The graph of which function shares a vertex with the graph of \( y = 5x^2 + 3 \)?
   A. \( y = 3x^2 + 5 \)
   B. \( y = x^2 + 3 \)
   C. \( y = (x + 3)^2 \)
   D. \( y = (x - 5)^2 \)

12. Identify the transformations from the graph of \( f(x) = x^2 \) to the graph of each function.
   a. \( y = 12x^2 \)
   b. \( y = \frac{2}{3}x^2 \)

13. Write the equation of the function whose graph has been transformed from the graph of the parent function as described.
   a. a vertical stretch by a factor of 6
   b. a vertical shrink by a factor of \( \frac{1}{2} \)

14. Write the equation of the quadratic function shown in the graph.

Lesson 30-3

15. Identify the transformations from the graph of the parent function \( f(x) = x^2 \) to the graph of each function. Then graph the function.
   a. \( y = 2x^2 + 1 \)
   b. \( y = -(x + 3)^2 \)
   c. \( y = (x + 1)^2 + 4 \)
   d. \( y = -\frac{1}{2}(x - 1)^2 + 1 \)

16. Write the equation of the quadratic function whose graph has the given vertex and passes through the given point.
   a. vertex: (0, \(-1\)); point (1, 0)
   b. vertex: (2, 0); point (0, 4)
   c. vertex: (\(-1, 5\)); point (2, \(-4\))

MATHEMATICAL PRACTICES
Construct Viable Arguments and Critique the Reasoning of Others

17. Explain why the domains of the functions \( y = x^2 \) and \( y = x^2 + c \), for any real number \( c \), are the same. Then write a statement about the ranges of the functions \( y = x^2 \) and \( y = (x + c)^2 \).
In 1680, Isaac Newton, scientist, astronomer, and mathematician, used a comet visible from Earth to prove that some comets follow a parabolic path through space as they travel around the sun. This and other discoveries like it help scientists to predict past and future positions of comets.

1. Assume the path of a comet is given by the function $y = -x^2 + 4$.
   a. Graph the path of the comet. Explain how you graphed it.
   b. Identify the vertex of the function.
   c. Identify the maximum or minimum value of the function.
   d. Identify the domain and range.
   e. Write the equation for the axis of symmetry.

2. Identify the table that represents a parabolic comet path. Explain and justify your choice.
   A. | x  | y  |
      |----|----|
      | -2 | -1 |
      | -1 | -4 |
      | 0  | -5 |
      | 1  | -4 |
      | 2  | -1 |
   B. | x  | y  |
      |----|----|
      | -2 |  4 |
      | -1 | -3 |
      | 0  | -5 |
      | 1  | -3 |
      | 2  |  4 |

3. The graph at right shows a portion of the path of a comet represented by a function in the form $y = a(x-h)^2 + k$. Determine the values of $a$, $h$, and $k$ and write the equation of the function.

4. The equation and graph below represent two different quadratic functions for the parabolic paths of comets.
   Function 1: $y = -3x^2 + 4$
   Function 2:
   
   Identify the maximum value of each function. Which function has the greater maximum value?
<table>
<thead>
<tr>
<th>Scoring Guide</th>
<th>Exemplary</th>
<th>Proficient</th>
<th>Emerging</th>
<th>Incomplete</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathematics Knowledge and Thinking (Items 2, 3)</td>
<td>• Clear and accurate understanding of how to determine whether a table of values represents a quadratic function</td>
<td>• Largely correct understanding of how to determine whether a table of values represents a quadratic function</td>
<td>• Difficulty determining whether a table of values represents a quadratic function</td>
<td>• Inaccurate or incomplete understanding of how to determine whether a table of values represents a quadratic function</td>
</tr>
<tr>
<td></td>
<td>• Effective understanding of quadratic functions as transformations of ( y = x^2 )</td>
<td>• Adequate understanding of quadratic functions as transformations of ( y = x^2 )</td>
<td>• Partial understanding of quadratic functions as transformations of ( y = x^2 )</td>
<td>• Little or no understanding of quadratic functions as transformations of ( y = x^2 )</td>
</tr>
<tr>
<td>Problem Solving (Items 2, 4)</td>
<td>• Appropriate and efficient strategy that results in a correct answer</td>
<td>• Strategy that may include unnecessary steps but results in a correct answer</td>
<td>• Strategy that results in a partially incorrect answer</td>
<td>• No clear strategy when solving problems</td>
</tr>
<tr>
<td>Mathematical Modeling / Representations (Items 1–4)</td>
<td>• Fluency in using tables, equations, and graphs to represent quadratic functions</td>
<td>• Little difficulty using tables, equations, and graphs to represent quadratic functions</td>
<td>• Some difficulty using tables, equations, and/or graphs to represent quadratic functions</td>
<td>• Significant difficulty using tables, equations, and/or graphs to represent quadratic functions</td>
</tr>
<tr>
<td></td>
<td>• Effective understanding of how to graph a quadratic function and identify key features from the graph</td>
<td>• Largely correct understanding of how to graph a quadratic function and identify key features from the graph</td>
<td>• Partial understanding of how to graph a quadratic function and/or identify key features from the graph</td>
<td>• Inaccurate or incomplete understanding of how to graph a quadratic function and/or identify key features from the graph</td>
</tr>
<tr>
<td>Reasoning and Communication (Items 1a, 2)</td>
<td>• Precise use of appropriate math terms and language to explain how to graph a quadratic function and whether a table of values represents a quadratic function</td>
<td>• Adequate explanation of how to graph a quadratic function and whether a table of values represents a quadratic function</td>
<td>• Misleading or confusing explanation of how to graph a quadratic function and/or whether a table of values represents a quadratic function</td>
<td>• Incomplete or inaccurate explanation of how to graph a quadratic function and/or whether a table of values represents a quadratic function</td>
</tr>
</tbody>
</table>
Learning Targets:
- Use a graph to solve a quadratic equation.
- Use factoring to solve a quadratic equation.
- Describe the connection between the zeros of a quadratic function and the x-intercepts of the function’s graph.

SUGGESTED LEARNING STRATEGIES: Visualization, Summarizing, Paraphrasing, Think-Pair-Share, Quickwrite

Carter, Alisha, and Joseph are building a trebuchet for an engineering competition. A trebuchet is a medieval siege weapon that uses gravity to launch an object through the air. When the counterweight at one end of the throwing arm drops, the other end rises and a projectile is launched through the air. The path the projectile takes through the air is modeled by a parabola.

To win the competition, the team must build their trebuchet according to the competition specifications to launch a small projectile as far as possible. After conducting experiments that varied the projectile’s mass and launch angle, the team discovered that the ball they were launching followed the path given by the quadratic equation \( y = -\frac{1}{8}x^2 + 2x \).

1. **Make sense of problems.** How far does the ball land from the launching point?
2. What is the maximum height of the ball?
3. What are the x-coordinates of the points where the ball is on the ground?
To determine how far the ball lands from the launching point, you can solve the equation $-\frac{1}{8}x^2 + 2x = 0$, because the height $y$ equals 0.

4. Verify that $x = 0$ and $x = 16$ are solutions to this equation.

5. Without the graph, could you have determined these solutions? Explain.

The factored form of a polynomial equation provides an effective way to determine the values of $x$ that make the equation equal 0.

**Zero Product Property**

If $ab = 0$, then either $a = 0$ or $b = 0$.

**Example A**

Solve $-\frac{1}{8}x^2 + 2x = 0$ by factoring.

**Step 1:** Factor.

$$x\left(-\frac{1}{8}x + 2\right) = 0$$

**Step 2:** Apply the Zero Product Property. $x = 0$ or $-\frac{1}{8}x + 2 = 0$

**Step 3:** Solve each equation for $x$.

$x = 0$ or $-\frac{1}{8}x = -2$

$$(-8)\left(-\frac{1}{8}\right)x = -2(-8)$$

$x = 16$

**Solution:** $x = 0$ or $x = 16$

**Try These A**

Solve each quadratic equation by factoring.

a. $x^2 - 5x - 14 = 0$

b. $3x^2 - 6x = 0$

c. $x^2 + 3x = 18$
Lesson 31-1
Solving by Graphing or Factoring

6. How do the solutions to the projectile path equation $-\frac{1}{8}x^2 + 2x = 0$ in Example A relate to the equation's graph?

The graph of the function $y = x^2 - 6x + 5$ is shown below.

7. Identify the $x$-intercepts of the graph.

8. What is the $x$-coordinate of the vertex?

9. Describe the $x$-coordinate of the vertex with respect to the two $x$-intercepts.

10. a. Factor the quadratic expression $x^2 - 6x + 5$.

   b. Each of the factors you found in part a is a linear expression. Solve the related linear equation for each factor by setting it equal to 0.

   c. Solve the related quadratic equation for the expression in Part a. That is, solve $x^2 - 6x + 5 = 0$.

11. a. How do the linear factors of $x^2 - 6x + 5$ you found in Item 10a relate to the $x$-intercepts of the graph of the function $y = x^2 - 6x + 5$ shown above?

   b. Without graphing, how could you determine the $x$-intercepts of the graph of the quadratic function $y = x^2 + x - 12$?
The $x$-coordinates of the $x$-intercepts of a quadratic function $y = ax^2 + bx + c$ are the **zeros of the function**. The solutions of a quadratic equation $ax^2 + bx + c = 0$ are the **roots** of the equation.

12. The quadratic function $y = ax^2 + bx + c$ is related to the equation $ax^2 + bx + c = 0$ by letting $y$ equal zero.
   a. Why do you think the $x$-coordinates of the $x$-intercepts are called the zeros of the function?
   b. Describe the relationship between the real roots of a quadratic equation and the zeros of the related quadratic function.

**Check Your Understanding**

Solve by factoring.

13. $x^2 - 81 = 0$
14. $\frac{1}{2}x^2 - 2x = 0$

Identify the zeros of the quadratic function. How are the linear factors of the quadratic expression related to the zeros?

15. $y = x^2 + 8x + 7$
16. $y = x^2 - 3x + 2$

**LESSON 31-1 PRACTICE**

17. **Make use of structure.** Use the graph of the quadratic function $y = 2x^2 + 6x$ shown to determine the roots of the quadratic equation $0 = 2x^2 + 6x$.

Solve by factoring.

18. $x^2 - 2x + 1 = 0$
19. $2x^2 - 7x - 4 = 0$
20. $x^2 + 5x = 0$

21. Write a quadratic equation whose roots are 3 and $-6$.

22. A whale jumps vertically from a pool at Ocean World. The function $y = -16x^2 + 32x$ models the height of a whale in feet above the surface of the water after $x$ seconds.
   a. What is the maximum height of the whale above the surface of the water?
   b. How long is the whale out of the water? Justify your answer.

23. **Construct viable arguments.** Is it possible for two different quadratic functions to share the same zeros? Use a graph to justify your response.
Lesson 31-2
The Axis of Symmetry and the Vertex

Learning Targets:

• Identify the axis of symmetry of the graph of a quadratic function.
• Identify the vertex of the graph of a quadratic function.

SUGGESTED LEARNING STRATEGIES: Close Reading, Marking the Text, Think-Pair-Share, Predict and Confirm, Discussion Groups

The axis of symmetry of the parabola determined by the function

\[ y = ax^2 + bx + c \]

is the vertical line that passes through the vertex.

The equation for the axis of symmetry is

\[ x = -\frac{b}{2a} \]

The vertex is on the axis of symmetry. Therefore, the x-coordinate of the vertex is

\[ -\frac{b}{2a} \]

Each point on a quadratic graph will have a mirror image point with the same y-coordinate that is equidistant from the axis of symmetry. For example, the point (0, 5) is reflected over the axis of symmetry to the point (6, 5) on the graph.

1. Give the coordinates of two additional points that are reflections over the axis of symmetry in the graph above.

2. For each function, identify the zeros graphically. Confirm your answer by setting the function equal to 0 and solving by factoring.

a. \[ y = x^2 + 2x - 8 \]  
b. \[ y = -x^2 + 4x + 5 \]
Lesson 31-2
The Axis of Symmetry and the Vertex

3. For each graph in Item 2, determine the $x$-coordinate of the vertex by finding the $x$-coordinate exactly in the middle of the two zeros. Confirm your answer by calculating the value of $-\frac{b}{2a}$.

a. $y = x^2 + 2x - 8$

b. $y = -x^2 + 4x + 5$

4. For each graph in Item 2, determine the $y$-coordinate of the vertex from the graph. Confirm your answer by evaluating the function at $x = -\frac{b}{2a}$.

a. $y = x^2 + 2x - 8$

b. $y = -x^2 + 4x + 5$

5. Reason quantitatively. For each graph in Item 2, determine the maximum or minimum value of the function.

a. $y = x^2 + 2x - 8$

b. $y = -x^2 + 4x + 5$

Check Your Understanding

For the function $y = -x^2 + x + 6$, use the value of $-\frac{b}{2a}$ to respond to the following.

6. Identify the vertex of the graph.

7. Write the equation of the axis of symmetry.
Lesson 31-2
The Axis of Symmetry and the Vertex

LESSON 31-2 PRACTICE
Use the value of $-\frac{b}{2a}$ to determine the vertex and to write the equation for the axis of symmetry for the graph of each of the following quadratic functions.

8. $y = x^2 - 10x$
9. $y = x^2 - 4x - 32$
10. $y = x^2 + x - 12$
11. $y = 6x - x^2$
12. Describe the graph of a quadratic function that has its vertex and a zero at the same point.

13. Model with mathematics. An architect is designing a tunnel and is considering using the function $y = -0.12x^2 + 2.4x$ to determine the shape of the tunnel's entrance, as shown in the figure. In this model, $y$ is the height of the entrance in feet and $x$ is the distance in feet from one end of the entrance.

![Graph of y = -0.12x^2 + 2.4x]

a. How wide is the tunnel's entrance at its base?
b. What is the vertex? What does it represent?
c. Could a truck that is 14 feet tall pass through the tunnel? Explain.
Learning Targets:

- Use the axis of symmetry, the vertex, and the zeros to graph a quadratic function.
- Interpret the graph of a quadratic function.

SUGGESTED LEARNING STRATEGIES: Create Representations, Close Reading, Note Taking, Identify a Subtask

If a quadratic function can be written in factored form, you can graph it by finding the vertex and the zeros.

**Example A**

Graph the quadratic function \( y = x^2 - x - 12 \).

**Step 1:** Determine the axis of symmetry using \( x = -\frac{b}{2a} \).

\[
\begin{align*}
\text{The axis of symmetry is } x &= 0.5. \\
&= \frac{-(-1)}{2(1)} = \frac{1}{2} = 0.5
\end{align*}
\]

**Step 2:** Determine the vertex.

The \( x \)-coordinate is 0.5.

Substitute 0.5 for \( x \) to find the \( y \)-coordinate.

The vertex is (0.5, -12.25).

**Step 3:** Determine the zeros of the function.

Set the function equal to 0 and solve.

\[
\begin{align*}
&\text{The zeros are } -3 \text{ and } 4. \\
x - 4 &= 0 \text{ or } x + 3 = 0 \\
&x = 4 \text{ or } x = -3
\end{align*}
\]

**Step 4:** Graph the function.

Plot the vertex (0.5, -12.25) and the points where the function crosses the \( x \)-axis (-3, 0) and (4, 0). Connect the points with a smooth parabolic curve.
Lesson 31-3
Graphing a Quadratic Function

Try These A

a. Check the graph in Example A by plotting two more points on the graph. First choose an x-value, and then find the y-value by evaluating the function. Plot this point. Then plot the reflection of the point over the axis of symmetry to get another point. Verify that both points are on the graph.

Graph the quadratic functions by finding the vertex and the zeros. Check your graphs.

b. \( y = x^2 - 2x - 8 \)
c. \( y = 4x - x^2 \)
d. \( y = x^2 + 4x - 5 \)

Joseph, Carter, and Alisha tested a new trebuchet designed to launch the projectile even further. They also refined their model to reflect a more accurate launch height of 1 m. The new projectile path is given by the function \( y = -\frac{1}{19}(x^2 - 18x - 19) \).

1. Graph the projectile path on the coordinate axes below.

CONNECT TO AP

In AP Calculus, you will represent projectile motion with special types of equations called parametric equations.
2. Make sense of problems. Last year’s winning trebuchet launched a projectile a horizontal distance of 19.5 m. How does the team’s trebuchet compare to last year’s winner?

Check Your Understanding

Graph each quadratic function.

3. \( y = x^2 - 11x + 30 \)
4. \( y = x^2 - 9 \)

LESSON 31-3 PRACTICE
Graph the quadratic functions. Label the vertex, axis of symmetry, and zeros on each graph.

5. \( y = x^2 + 5x \)
6. \( y = -x^2 + 2x \)
7. \( y = -x^2 + \frac{1}{4} \)
8. \( y = x^2 - 3x - 4 \)
9. \( y = -x^2 + 3x + 4 \)
10. Attend to precision. Describe the similarities and differences between the graphs of the functions in Items 8 and 9. How are these similarities and differences indicated by the functions themselves?
ACTIVITY 31 PRACTICE
Write your answers on notebook paper. Show your work.

Lesson 31-1
Use the graphs to determine the zeros of the quadratic functions.

1. \( y = 0.5x^2 \)

2. \( y = x^2 - 4 \)

3. \( y = -x^2 + 4x - 3 \)

4. Identify the zeros of the quadratic function \( y = (x - a)(x - b) \).

5. Which of the following functions has zeros at \( x = 2 \) and \( x = -3 \)?
   A. \( y = x^2 + x - 6 \)  
   B. \( y = -x^2 + 5x + 6 \)  
   C. \( y = 2x^2 + x - 3 \)  
   D. \( y = x^2 + 2x - 3 \)  

6. What are the solutions to the equation \( x^2 - 6x = -5 \)?
   A. \( -1, 5 \)  
   B. \( 2, 3 \)  
   C. \( 1, 5 \)  
   D. \( -2, 3 \)  

Solve by factoring.

7. \( 0 = x^2 - 25 \)

8. \( 0 = x^2 + 17x \)

9. \( 0 = x^2 - 5x - 66 \)

10. \( 0 = x^2 + 99x - 100 \)

11. \( 0 = -x^2 - 4x + 32 \)

12. \( 0 = x^2 + 10x + 25 \)

13. \( 0 = x^2 + 8x + 7 \)

14. A fountain at a city park shoots a stream of water vertically from the ground. The function \( y = -8x^2 + 16x \) models the height of the stream of water in feet after \( x \) seconds.
   a. What is the maximum height of the stream of water?
   b. At what time does the stream of water reach its maximum height?
   c. For how many seconds does the stream of water appear above ground?
   d. Identify a reasonable domain and range for this function.

15. Write a quadratic equation whose roots are \(-4\) and \(8\). How do the roots relate to the zeros and factors of the associated quadratic function?

16. Write a quadratic function whose zeros are \(2\) and \(9\). How do the zeros relate to the factors of the quadratic function?
Lesson 31-2

17. If a quadratic function has zeros at \( x = -4 \) and \( x = 6 \), what is the \( x \)-coordinate of the vertex?

18. What is the vertex of the quadratic function \( y = x^2 + 8x + 15 \)?

19. Write the equation for the axis of symmetry of the graph of the quadratic function \( y = 2x^2 + 4x - 1 \).

Use the following information for Items 20–24.

A diver in Acapulco jumps from a cliff. His height \( y \), in meters, as a function of \( x \), his distance from the cliff base in meters, is given by the quadratic function \( y = 100 - x^2 \), for \( x \geq 0 \).

20. Graph the function representing the cliff diver’s height.
   a. Identify the vertex of the graph
   b. Identify the \( x \)-intercepts of the graph.

21. Determine the solutions of the equation \( 100 - x^2 = 0 \).

22. How do the solutions to the equation in Item 21 relate to the \( x \)-intercepts of the graph in Item 20?

23. How high is the cliff from which the diver jumps?

24. How far from the base of the cliff does the diver hit the water?

25. Which of these functions has a graph with the axis of symmetry \( x = -2 \)?
   A. \( y = x^2 + 4x - 2 \)
   B. \( y = x^2 - 4x + 2 \)
   C. \( y = 2x^2 + 2x - 3 \)
   D. \( y = 2x^2 - 2x + 3 \)

Lesson 31-3

26. Lisa correctly graphed a quadratic function and found that its vertex was in Quadrant I. Which function could she have graphed?
   A. \( y = x^2 + 4x + 2 \)
   B. \( y = x^2 - 4x + 2 \)
   C. \( y = x^2 - 4x + 6 \)
   D. \( y = x^2 + 4x + 6 \)

27. Which of these is the equation of the parabola graphed below?
   ![Graph of a parabola]
   A. \( y = 2x^2 - 2x + 8 \)
   B. \( y = -2x^2 + 8 \)
   C. \( y = -x^2 - 8x - 15 \)
   D. \( y = -x^2 - 8 \)

28. DeShawn’s textbook shows the graph of the function \( y = x^2 + x - 6 \). Which of these is a true statement about the graph?
   A. The axis of symmetry is the \( y \)-axis.
   B. The vertex is \((0, -6)\).
   C. The graph intersects the \( x \)-axis at \((-3, 0)\).
   D. The graph is a parabola that opens downward.

29. Kim throws her basketball up from the ground toward the basketball hoop from a distance of 20 feet away from the hoop. The ball follows a parabolic path and returns back to the gym floor 5 feet from the hoop. Write one possible equation to represent the path of the basketball. Explain your answer.

MATHEMATICAL PRACTICES

Use Appropriate Tools Strategically

30. Consider the quadratic function \( y = x^2 - x - 3 \).
   a. Is it possible to find the zeros of the function by factoring? Explain.
   b. Use your calculator to graph the function.
      Based on the graph, what are the approximate zeros of the function?
   c. Use the zero function of your calculator to find more accurate approximations for the zeros. Round to the nearest tenth.
Learning Targets:
- Solve quadratic equations by the square root method.
- Provide examples of quadratic equations having a given number of real solutions.

SUGGESTED LEARNING STRATEGIES: Guess and Check, Simplify the Problem, Think-Pair-Share, Create Representations

Nguyen is trying to build a square deck around his new hot tub. To decide how large a deck he should build, he needs to determine the side length, \( x \), of different sized decks given the possible area of each deck. He knows that the area of a square is equal to the length of a side squared. Using this information, Nguyen writes the following equations to represent each of the decks he is considering.

1. Solve each equation. Be prepared to discuss your solution methods with your classmates.
   - a. \( x^2 = 49 \)
   - b. \( x^2 = 100 \)
   - c. \( x^2 = 15 \)
   - d. \( 2x^2 = 18 \)
   - e. \( x^2 - 4 = 0 \)
   - f. \( x^2 + 2 = 0 \)
   - g. \( x^2 + 3 = 3 \)

2. Refer to the equations in Item 1 and their solutions.
   - a. What do the equations have in common?
   - b. What types of numbers are represented by the solutions of these equations?
   - c. How many solutions do the equations have?
   - d. Reason quantitatively. Which solutions are reasonable for side lengths of the squares? Explain.

A solution of an equation makes the equation true. For example, \( x + 5 = 7 \) has the solution \( x = 2 \) because \( 2 + 5 = 7 \).
To solve a quadratic equation of the form \(ax^2 + c = 0\), isolate the \(x^2\)-term and then take the square root of both sides.

**Example A**
Solve \(3x^2 - 6 = 0\) using square roots.

**Step 1:** Add 6 to both sides. \(3x^2 = 6\)

**Step 2:** Divide both sides by 3. \(\frac{3x^2}{3} = \frac{6}{3}\)

\[x^2 = 2\]

**Step 3:** Take the square root of both sides. \(\sqrt{x^2} = \pm\sqrt{2}\)

**Solution:** \(x = +\sqrt{2}\) or \(x = -\sqrt{2}\)

**Try These A**
Solve each equation using square roots.

a. \(x^2 - 10 = 1\)

b. \(\frac{x^2}{4} = 1\)

c. \(4x^2 - 6 = 14\)

3. Quadratic equations can have 0, 1, or 2 real solutions. Fill in the table below with equations from the first page that represent the possible numbers of solutions.

<table>
<thead>
<tr>
<th>Number of Solutions</th>
<th>Result When (x^2) is Isolated</th>
<th>Example(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Two</td>
<td>(x^2 = \text{positive number})</td>
<td></td>
</tr>
<tr>
<td>One</td>
<td>(x^2 = 0)</td>
<td></td>
</tr>
<tr>
<td>No real solutions</td>
<td>(x^2 = \text{negative number})</td>
<td></td>
</tr>
</tbody>
</table>
Lesson 32-1
The Square Root Method

4. **Reason abstractly.** A square frame has a 2-in. border along two sides as shown in the diagram. The total area is 66 in². Answer the questions to help you write an equation to find the area of the unshaded square.

![Diagram of a square frame with a 2-in. border along two sides.]

a. Label the sides of the unshaded square \(x\).
b. Fill in the boxes to write an equation for the total area in terms of \(x\).

\[
\text{Area in terms of } x = \square \text{ Area in square in.}
\]

You can solve quadratic equations like the one you wrote in Item 4 by isolating the variable.

**Example B**
Solve \((x + 2)^2 = 66\) using square roots. Approximate the solutions to the nearest hundredth.

**Step 1:** Take the square root of both sides.

\[
\sqrt{(x + 2)^2} = \pm \sqrt{66}
\]

\[
x + 2 = \pm \sqrt{66}
\]

**Step 2:** Subtract 2 from both sides.

\[
x = -2 \pm \sqrt{66}
\]

\[
x = -2 + \sqrt{66} \text{ or } x = -2 - \sqrt{66}
\]

**Step 3:** Use a calculator to approximate the solutions.

**Solution:** \(x \approx -10.12\) or \(x \approx 6.12\)

5. Are both solutions to this equation valid in the context of Item 4? Explain your response.

**Try These B**
Solve each equation using square roots.

a. \((x - 5)^2 = 121\)  
b. \((2x - 1)^2 = 6\)  
c. \(x^2 - 12x + 36 = 2\)
Lesson 32-1
The Square Root Method

Check Your Understanding

Solve each equation using square roots.

6. \( x^2 + 12 = 13 \)  
7. \( (x - 4)^2 = 1 \)  
8. \( 3x^2 - 6 = 15 \)

9. Give an example of a quadratic equation that has
   a. one real solution.
   b. no real solutions.
   c. two real solutions.

LESSON 32-1 PRACTICE

10. If the length of a square is decreased by 1 unit, the area will be 8 square units. Write an equation for the area of the square.

11. Calculate the side length of the square in Item 10.

Solve each equation.

12. \( x^2 - 22 = 0 \)  
13. \( (x + 5)^2 - 4 = 0 \)
14. \( x^2 - 4x + 4 = 0 \)  
15. \( (x + 1)^2 = 12 \)

16. Model with mathematics. Alaysha has a square picture with an area of 100 square inches, including the frame. The width of the frame is \( x \) inches.

a. Write an equation in terms of \( x \) for the area \( A \) of the picture inside the frame.

b. If the area of the picture inside the frame is 64 square inches, what are the possible values for \( x \)?

c. If the area of the picture inside the frame is 64 square inches, how wide is the picture frame? Justify your response.
Learning Targets:
- Solve quadratic equations by completing the square.
- Complete the square to analyze a quadratic function.

SUGGESTED LEARNING STRATEGIES: Note Taking, Graphic Organizer, Identify a Subtask

As shown in Example B in Lesson 32-1, quadratic equations are more easily solved with square roots when the side with the variable is a perfect square. When a quadratic equation is written in the form \(x^2 + bx + c = 0\), you can complete the square to transform the equation into one that can be solved using square roots. **Completing the square** is the process of adding a term to the variable side of a quadratic equation to transform it into a perfect square trinomial.

**Example A**
Solve \(x^2 + 10x - 6 = 0\) by completing the square.

**Step 1:** Isolate the variable terms.
Isolate the variable terms. \(x^2 + 10x - 6 = 0\)
Add 6 to both sides. \(x^2 + 10x = 6\)

**Step 2:** Transform the left side into a perfect square trinomial.
Divide the coefficient of the \(x\)-term by 2. \(10 \div 2 = 5\)
Square the 5 to determine the constant. \(5^2 = 25\)
Complete the square by adding 25 to both sides of the equation. \(x^2 + 10x + 25 = 6 + 25\)

**Step 3:** Solve the equation.
Write the trinomial in factored form. \((x + 5)(x + 5) = 31\)
Write the left side as a square of a binomial. \((x + 5)^2 = 31\)
Take the square root of both sides. \(\sqrt{(x + 5)^2} = \pm\sqrt{31}\)
Solve for \(x\). \(x + 5 = \pm\sqrt{31}\)
Leave the solutions in ± form. \(x = -5 \pm \sqrt{31}\)

**Solution:** \(x = -5 \pm \sqrt{31}\)

**Try These A**
**Make use of structure.** Solve each quadratic equation by completing the square.

\[a. \ x^2 - 8x + 3 = 11 \quad b. \ x^2 + 7 = 2x + 8\]
Completing the square is useful to help analyze specific features of quadratic functions, such as the maximum or minimum value and the possible number of zeros.

When a quadratic equation is written in the form $y = a(x - h)^2 + k$, you can determine whether the function has a maximum or minimum value based on $a$ and what that value is based on $k$. This information can also help you determine the number of $x$-intercepts.

**Example B**

Analyze the quadratic function $y = x^2 - 6x + 13$ by completing the square.

**Step 1:** Complete the square on the right side of the equation.

$y = x^2 - 6x + 13$

Isolate the variable terms.

$y - 13 = x^2 - 6x$

Transform the right side into a perfect square trinomial.

$y - 13 + 9 = x^2 - 6x + 9$

Factor and simplify.

$y - 4 = (x - 3)^2$

Write the equation in the form $y = a(x - h)^2 + k$.

$y = (x - 3)^2 + 4$

**Step 2:** Identify the direction of opening and whether the vertex represents a maximum or minimum.

Since the value of $a$ is positive, the parabola opens upward and the vertex represents a minimum.

**Step 3:** Determine the maximum or minimum value. This parabola is the graph of the parent function $y = x^2$ translated up 4 units. So the parent function's minimum value of 0 is increased to 4.

**Step 4:** Determine the number of $x$-intercepts.

Since the minimum value of the function is $y = 4$ and the parabola opens upward, the function will never have a $y$-value less than 4. The graph will never intersect the $x$-axis, so there are no $x$-intercepts.

**Step 5:** Verify by graphing.

**Solution:** The graph of the quadratic function opens upward, the minimum value is 4, and there are no $x$-intercepts.
Lesson 32-2
Completing the Square

Try These B
Write each of the following quadratic functions in the form
\( y = a(x - h)^2 + k \). Identify the direction of opening, vertex, maximum or minimum value, and number of \( x \)-intercepts.

a. \( y = x^2 - 4x + 9 \)

b. \( y = -x^2 - 6x - 8 \)

c. \( y = x^2 + 8x + 15 \)

Check Your Understanding

Solve by completing the square.

1. \( x^2 + 2x + 3 = 0 \)  
2. \( x^2 + 6x + 4 = 0 \)

LESSON 32-2 PRACTICE

Solve by completing the square.

3. \( 2 = x^2 - 10x \)  
4. \( 4x = x^2 - 4x - 32 \)  
5. \( -2x^2 + 4 = -x^2 + x - 7 \)  
6. \( x + 1 = 6x - x^2 \)

Complete the square to determine the vertex and maximum or minimum value. Determine the number of \( x \)-intercepts.

7. \( y = x^2 - 2x + 2 \)  
8. \( y = -x^2 + 8x - 6 \)

9. Make sense of problems. In a model railroad, the track is supported by an arch that is represented by \( y = -x^2 + 10x - 16 \), where \( y \) represents the height of the arch in inches and \( x \) represents the distance in inches from a cliff. Complete the square to answer the following questions.
   a. How far is the center of the arch from the cliff?
   b. What is the maximum height of the arch?

10. The bubbler is the part of a drinking fountain that produces a stream of water. The water in a drinking fountain follows a path given by \( y = -x^2 + 6x + 4.5 \), where \( y \) is the height of the water in centimeters above the basin, and \( x \) is the distance of the water from the bubbler. What is the maximum height of the water above the basin?
Learning Targets:

• Derive the quadratic formula.
• Solve quadratic equations using the quadratic formula.

**SUGGESTED LEARNING STRATEGIES:** Close Reading, Note Taking, Identify a Subtask

Generalizing a solution method into a formula provides an efficient way to perform complicated procedures. You can complete the square on the general form of a quadratic equation $ax^2 + bx + c = 0$ to find a formula for solving all quadratic equations.

$$ax^2 + bx + c = 0$$

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$$

$$x + \frac{b}{2a} = \pm \sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$x = -\frac{b \pm \sqrt{b^2 - 4ac}}{2a}$$

**Quadratic Formula**

When $a \neq 0$, the solutions of $ax^2 + bx + c = 0$ are

$$x = -\frac{b \pm \sqrt{b^2 - 4ac}}{2a}.$$
Lesson 32-3
The Quadratic Formula

To apply the quadratic formula, make sure the equation is in standard form
\[ ax^2 + bx + c = 0. \]
Identify the values of \( a \), \( b \), and \( c \) in the equation and then substitute these values into the quadratic formula. If the expression under the radical sign is not a perfect square, write the solutions in simplest radical form or use a calculator to approximate the solutions.

Example A
Solve \( x^2 + 3 = 6x \) using the quadratic formula.

Step 1: Write the equation in standard form.
\[
 x^2 + 3 = 6x \\
 x^2 - 6x + 3 = 0
\]

Step 2: Identify \( a \), \( b \), and \( c \).
\( a = 1 \), \( b = -6 \), \( c = 3 \)

Step 3: Substitute these values into the quadratic formula.
\[
x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(3)}}{2(1)}
\]

Step 4: Simplify using the order of operations.
\[
x = \frac{6 \pm \sqrt{36 - 12}}{2} = \frac{6 \pm \sqrt{24}}{2}
\]

Step 5: Write as two solutions.
\[
x = \frac{6 + \sqrt{24}}{2} \text{ or } x = \frac{6 - \sqrt{24}}{2}
\]

Solution: Use a calculator to approximate the two solutions.
\( x \approx 5.45 \) or \( x \approx 0.55 \)

If you do not have a calculator, write your solution in simplest radical form.
To write the solution in simplest form, simplify the radicand and then divide out any common factors.
\[
x = \frac{6 \pm \sqrt{24}}{2} = \frac{6 \pm \sqrt{4 \cdot 6}}{2} = \frac{6 \pm 2\sqrt{6}}{2} = 3 \pm \sqrt{6}
\]

Try These A
Solve using the quadratic formula.

a. \( 3x^2 = 4x + 3 \)

b. \( x^2 + 4x = -2 \)
Lesson 32-3
The Quadratic Formula

Check Your Understanding

Solve using the quadratic formula.
1. \(3x^2 - 5x + 1 = 0\)
2. \(x^2 + 6 = -8x + 12\)

LESSON 32-3 PRACTICE

Solve using the quadratic formula.
3. \(x^2 + 5x - 1 = 0\)
4. \(-2x^2 - x + 4 = 0\)
5. \(4x^2 - 5x - 2 = 1\)
6. \(x^2 + 3x = -x + 1\)
7. \(3x^2 = -6x + 4\)
8. A baseball player tosses a ball straight up into the air. The function \(y = -16x^2 + 30x + 5\) models the motion of the ball, where \(x\) is the time in seconds and \(y\) is the height of the ball, in feet.
   a. Write an equation you can solve to find out when the ball is at a height of 15 feet.
   b. Use the quadratic formula to solve the equation. Round to the nearest tenth.
   c. How many solutions did you find for Part (b)? Explain why this makes sense.

9. Critique the reasoning of others. José and Marta each solved \(x^2 + 4x = -3\) using two different methods. Who is correct and what is the error in the other student’s work?

José
\[
\begin{align*}
x^2 + 4x &= -3 \\
x^2 + 4x - 3 &= 0 \\
a &= 1, \ b = 4, \ c = -3 \\
x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
&= \frac{-4 \pm \sqrt{4^2 - 4(1)(-3)}}{2(1)} \\
&= \frac{-4 \pm \sqrt{16 + 12}}{2} \\
&= \frac{-4 \pm \sqrt{28}}{2} \\
&= \frac{-4 \pm 2\sqrt{7}}{2} = -2 \pm \sqrt{7}
\end{align*}
\]

Marta
\[
\begin{align*}
x^2 + 4x &= -3 \\
x^2 + 4x + 3 &= -3 + 3 \\
x^2 + 4x + 4 &= -3 + 4 \\
(x + 2)^2 &= 1 \\
x + 2 &= \pm 1 \\
x &= -1 \text{ or } x = -3
\end{align*}
\]
Lesson 32-4
Choosing a Method and Using the Discriminant

Learning Targets:
• Choose a method to solve a quadratic equation.
• Use the discriminant to determine the number of real solutions of a quadratic equation.

SUGGESTED LEARNING STRATEGIES: Think-Pair-Share, Graphic Organizer, Look for a Pattern, Create a Plan, Quickwrite

There are several methods for solving a quadratic equation. They include factoring, using square roots, completing the square, and using the quadratic formula. Each of these techniques has different advantages and disadvantages. Learning how and why to use each method is an important skill.

1. Solve each equation below using a different method. State the method used.
   a. \( x^2 + 5x - 24 = 0 \)
   b. \( x^2 - 6x + 2 = 0 \)
   c. \( 2x^2 + 3x - 5 = 0 \)
   d. \( x^2 - 100 = 0 \)

2. How did you decide which method to use for each equation in Item 1?
The expression $\sqrt{b^2 - 4ac}$ in the quadratic formula helps you understand the nature of the quadratic equation. The discriminant, $b^2 - 4ac$, of a quadratic equation gives information about the number of real solutions, as well as the number of $x$-intercepts of the related quadratic function.

3. Solve each equation using any appropriate solution method. Then complete the rest of the table.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Discriminant</th>
<th>Solutions</th>
<th>Number of Real Solutions</th>
<th>Number of $x$-Intercepts</th>
<th>Graph of Related Quadratic Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x^2 + 2x - 8 = 0$</td>
<td>$-4$</td>
<td></td>
<td></td>
<td></td>
<td><img src="image1" alt="Graph" /></td>
</tr>
<tr>
<td>$x^2 + 2x + 1 = 0$</td>
<td>$-3$</td>
<td></td>
<td></td>
<td></td>
<td><img src="image2" alt="Graph" /></td>
</tr>
<tr>
<td>$x^2 + 2x + 5 = 0$</td>
<td>$-6$</td>
<td></td>
<td></td>
<td></td>
<td><img src="image3" alt="Graph" /></td>
</tr>
</tbody>
</table>
Lesson 32-4
Choosing a Method and Using the Discriminant

4. **Express regularity in repeated reasoning.** Complete each statement below using the information from the table in Item 3.

- If \( b^2 - 4ac > 0 \), the equation has ______ real solution(s) and the graph of the related function has ______ x-intercept(s).
- If \( b^2 - 4ac = 0 \), the equation has ______ real solution(s) and the graph of the related function has ______ x-intercept(s).
- If \( b^2 - 4ac < 0 \), the equation has ______ real solution(s) and the graph of the related function has ______ x-intercept(s).

**Check Your Understanding**

Use the discriminant to determine the number of real solutions.

5. \( 4x^2 + 2x - 12 = 0 \)
6. \( x^2 - 7x + 14 = 0 \)
7. \( x^2 - 10x + 25 = 0 \)

**LESSON 32-4 PRACTICE**

For each equation, use the discriminant to determine the number of real solutions. Then solve the equation.

8. \( x^2 - 1 = 0 \)
9. \( x^2 - 4x + 4 = 0 \)
10. \( -4x^2 + 3x = -2 \)
11. \( x^2 - 2 = 12x \)
12. \( x^2 + 5x - 1 = 0 \)
13. \( -x^2 - 2x - 10 = 0 \)
14. **Model with mathematics.** Lin launches a model rocket that follows a path given by the function \( y = -0.4t^2 + 3t + 0.5 \), where \( y \) is the height in meters and \( t \) is the time in seconds.
   a. Explain how you can write an equation and then use the discriminant to determine whether Lin’s rocket ever reaches a height of 5 meters.
   b. If Lin’s rocket reaches a height of 5 meters, at approximately what time(s) does it do so? If not, what is the rocket’s maximum height?
Learning Targets:

- Use the imaginary unit \( i \) to write complex numbers.
- Solve a quadratic equation that has complex solutions.

**SUGGESTED LEARNING STRATEGIES:** Think-Pair-Share, Close Reading, Note Taking, Construct an Argument, Identify a Subtask

When solving quadratic equations, there are always one, two, or no real solutions. Graphically, the number of \( x \)-intercepts is helpful for determining the number of real solutions.

- When there is one real solution, the graph of the related quadratic function touches the \( x \)-axis once, and the vertex of the parabola is on the \( x \)-axis.
- When there are two real solutions, the graph crosses the \( x \)-axis twice.
- When there are no real solutions, the graph never crosses the \( x \)-axis.

1. Graph the function \( y = x^2 - 6x + 13 \). Use the graph to determine the number of real solutions to the equation \( x^2 - 6x + 13 = 0 \).

2. Construct viable arguments. What does the number of real solutions to the equation in Item 1 indicate about the value of the discriminant of the equation? Explain.

When the value of the discriminant is less than zero, there are no real solutions. This is different from stating there are no solutions. In cases where the discriminant is negative, there are two solutions that are not real numbers. **Imaginary numbers** offer a way to determine these non-real solutions. The **imaginary unit**, \( i \), equals \( \sqrt{-1} \). Imaginary numbers are used to represent square roots of negative numbers, such as \( \sqrt{-4} \).
Lesson 32-5  
Complex Solutions

Example A  
Simplify \(-\sqrt{4}\).  

Step 1: Write the radical as a product involving \(-1\).  
\[ \sqrt{-4} = \sqrt{-1} \cdot \sqrt{4} \]

Step 2: Replace \(-1\) with the imaginary unit, \(i\).  
\[ = i\sqrt{4} \]

Step 3: Simplify the radical. The principal square root of 4 is 2.  
\[ = 2i \]

Solution: \(-\sqrt{4} = 2i\)

Try These A  
Simplify.  
a. \(-\sqrt{16}\)  
b. \(-\sqrt{9}\)  
c. \(-\sqrt{8}\)

Problems involving imaginary numbers can also result in complex numbers, \(a + bi\), where \(a\) and \(b\) are real numbers. In this form, \(a\) is the real part and \(b\) is the imaginary part.

Example B  
Solve \(x^2 - 6x + 12 = 0\).  

Step 1: Identify \(a\), \(b\), and \(c\).  
\(a = 1, b = -6, c = 12\)

Step 2: Substitute these values into the quadratic formula.  
\[ x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(12)}}{2(1)} \]

Step 3: Simplify using the order of operations.  
\[ x = \frac{6 \pm \sqrt{36 - 48}}{2} \]

Step 4: Simplify using the imaginary unit, \(i\).  
\[ x = \frac{6 \pm i\sqrt{12}}{2} \]

Step 5: Simplify the radical and the fraction.  
\[ x = \frac{6 \pm 2i\sqrt{3}}{2} = 3 \pm i\sqrt{3} \]

Solution: \(x = 3 \pm i\sqrt{3}\)

Try These B  
Solve each equation.  
a. \(x^2 + 100 = 0\)  
b. \(x^2 - 4x = -11\)
LESSON 32-5 PRACTICE

Simplify.
5. \(-\sqrt{-11}\)
6. \(\sqrt{-42}\)
7. \(\pm \sqrt{-81}\)

Solve.
8. \(5x^2 - 2x + 3 = 0\)
9. \(-x^2 - 6 = 0\)
10. \((x - 1)^2 + 3 = 0\)

11. Make use of structure. Consider the quadratic function
\(y = x^2 + 2x + c\), where \(c\) is a real number.
   a. Write and simplify an expression for the discriminant.
   b. Explain how you can use your result from Part (a) to write and solve an inequality that tells you when the function will have two zeros that involve imaginary numbers.
   c. Use your results to describe the zeros of the function \(y = x^2 + 2x + 3\).
ACTIVITY 32 PRACTICE
Write your answers on notebook paper. Show your work.

Lesson 32-1
Solve each equation using square roots.
1. \( x^2 + 7 = 43 \)
2. \( (x - 5)^2 + 2 = 11 \)
3. \( x^2 - 8x + 16 = 3 \)
4. Antonio drops a rock from a cliff that is 400 feet high. The function \( y = -16t^2 + 400 \) gives the height of the rock in feet after \( t \) seconds. Write and solve an equation to determine how long it takes the rock to land at the base of the cliff. (Hint: At the base of the cliff, the height \( y \) is 0.)
5. Maya wants to use square roots to solve the equation \( x^2 - 6x + 9 = k \), where \( k \) is a positive real number. Which of these is the best representation of the solution?
   A. \( x = 3 \pm \sqrt{k} \)
   B. \( x = -3 \pm \sqrt{k} \)
   C. \( x = \pm \sqrt{k+3} \)
   D. \( x = \pm \sqrt{k-3} \)

Lesson 32-2
6. Given the equation \( x^2 - 8x = 3 \), what number should be added to both sides to complete the square?
   A. \(-4\)
   B. \(8\)
   C. \(16\)
   D. \(64\)

Write each of the following equations in the form \( y = a(x - h)^2 + k \). Then identify the direction of opening, vertex, maximum or minimum value, and \( x \)-intercepts.
7. \( y = x^2 - 4x + 11 \)
8. \( y = -x^2 - 6x - 8 \)
9. \( y = x^2 + 2x - 8 \)

10. A golfer stands on a platform 16 feet above a driving range. Once the golf ball is hit, the function \( y = -16t^2 + 64t + 16 \) represents the height of the ball in feet after \( t \) seconds.
   a. Write an equation you can solve to determine the number of seconds it takes for the ball to land on the driving range.
   b. Solve the equation by completing the square. Leave your answer in radical form.
   c. Use a calculator to find the number of seconds, to the nearest tenth, that it takes the ball to land on the driving range.

11. Which of the following is a true statement about the graph of the quadratic function \( y = x^2 - 2x + 3 \)?
   A. The vertex of the graph is \((-1, 2)\).
   B. The graph intersects the \( x \)-axis at \( x = 1 \).
   C. The graph is a parabola that opens upward.
   D. There is exactly one \( x \)-intercept.

Solve by completing the square.
12. \( x^2 - 4x = 12 \)
13. \( x^2 + 10x + 21 = 0 \)
14. \( 2x^2 - 4x - 4 = 0 \)
15. \( x^2 + 6x = -10 \)
16. A climbing structure at a playground is represented by the function \( y = -x^2 + 4x + 1 \), where \( y \) is the height of the structure in feet and \( x \) is the distance in feet from a wall. What is the maximum height of the structure?
   A. 1 foot
   B. 2 feet
   C. 4 feet
   D. 5 feet
Lesson 32-3

Solve using the quadratic formula.

17. $4x^2 - 4x = 3$
18. $5x^2 - 9x - 2 = 0$
19. $x^2 = 2x + 4$
20. A football player kicks a ball. The function $y = -16t^2 + 32t + 3$ models the motion of the ball, where $t$ is the time in seconds and $y$ is the height of the ball in feet.
   a. Write an equation you can solve to find out when the ball is at a height of 11 feet.
   b. Use the quadratic formula to solve the equation. Round to the nearest tenth.
21. Kyla was asked to solve the equation $2x^2 + 6x - 1 = 0$. Her work is shown below.
   Is her solution correct? If not, describe the error and give the correct solution.

   
   
   
   
   
   
   

Lesson 32-4

Use the discriminant to determine the number of real solutions.

22. $x^2 + 3x + 5 = 0$
23. $4x^2 - 4x + 1 = 0$
24. The discriminant of a quadratic equation is $-1$. Which of the following must be a true statement about the graph of the related quadratic function?
   A. The graph intersects the x-axis in exactly two points.
   B. The graph lies entirely above the x-axis.
   C. The graph intersects the x-axis at $x = -1$.
   D. The graph has no x-intercepts.
25. A dolphin jumps straight up from the water. The quadratic function $y = -16t^2 + 20t$ models the motion of the dolphin, where $t$ is the time in seconds and $y$ is the height of the dolphin, in feet. Use the discriminant to explain why the dolphin does not reach a height of 7 feet.

Lesson 32-5

Simplify.

26. $\pm \sqrt{-2}$
27. $-\sqrt{-25}$
28. $\sqrt{-8}$
29. $-\sqrt{-121}$
30. $12 - \sqrt{-144}$
31. $\pm \sqrt{-32}$

Solve.

32. $2x^2 - 5x + 5 = 0$
33. $x^2 + x + 3 = 0$
34. $-3x^2 - 3x - 1 = 0$
35. $-x^2 - x - 2 = 0$

MATHEMATICAL PRACTICES

Construct Viable Arguments and Critique the Reasoning of Others

36. For what values of $p$ does the quadratic function $y = x^2 + 4x + p$ have two real zeros? Justify your answer.
Cooper is a model rocket fan. Cooper’s model rockets have single engines and, when launched, can rise as high as 1000 ft depending upon the engine size. After the engine is ignited, it burns for 3–5 seconds, and the rocket accelerates upward. The rocket has a parachute that will open as the rocket begins to fall back to Earth.

Cooper wanted to investigate the flight of a rocket from the time the engine burns out until the rocket lands. He set a device in a rocket, named Spirit, to begin collecting data the moment the engine shut off. Unfortunately, the parachute failed to open. When the rocket began to descend, it was in free fall.

The table shows the data that was collected.

<table>
<thead>
<tr>
<th>Time Since the Engine Burned Out (s)</th>
<th>Height (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>512</td>
</tr>
<tr>
<td>1</td>
<td>560</td>
</tr>
<tr>
<td>2</td>
<td>576</td>
</tr>
<tr>
<td>3</td>
<td>560</td>
</tr>
<tr>
<td>4</td>
<td>512</td>
</tr>
<tr>
<td>5</td>
<td>432</td>
</tr>
<tr>
<td>6</td>
<td>320</td>
</tr>
</tbody>
</table>

1. a. Use the table to determine whether the height of the Spirit can be modeled by a linear function.

b. Graph the data for the height of the Spirit as a function of time on the grid.
2. Use the table and graph from Item 1.
   a. How high was the Spirit when the engine burned out?
   b. How long did it take the rocket to reach its maximum height after the engine cut out?
   c. Estimate the time the rocket was in free fall before it reached the earth.

3. Use the table and graph from Item 1.
   a. Use appropriate tools strategically. Use a graphing calculator to determine a quadratic function \( h(t) \) for the data.
   b. Sketch the graph of the function on the grid in Item 1.
   c. Attend to precision. Give a reasonable domain and range for \( h(t) \) within the context of the problem. Be sure to include units.

4. Use the function found in Item 3 to verify the height of the Spirit when the engine burned out.

5. Construct viable arguments. Use the graph of \( h(t) \) to approximate the time interval in which the Spirit was in free fall. Explain how you determined your answer.
Lesson 33-1
Fitting Data with a Quadratic Function

Check Your Understanding

Use the data in the table for Items 6–8.

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>f(x)</td>
<td>5</td>
<td>4.5</td>
<td>3</td>
<td>0.5</td>
</tr>
</tbody>
</table>

6. Graph the data in the table. Do the data appear to be linear? Explain.
7. Use a graphing calculator to write a function to model \( f(x) \).
8. Use the function to determine the value of \( f(x) \) when \( x = 1.5 \).

LESSON 33-1 PRACTICE

9. Model with mathematics. Cooper wanted to track another one of his rockets, the Eagle, so that he could investigate its time and height while in flight. He installed a device into the nose of the Eagle to measure the time and height of the rocket as it fell back to Earth. The device started measuring when the parachute opened. The data for one flight of the Eagle is shown in the table below.

<table>
<thead>
<tr>
<th>The Eagle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time Since Parachute Opened (s)</td>
</tr>
<tr>
<td>Height(ft)</td>
</tr>
</tbody>
</table>

a. Graph the data from the table.
b. Use a graphing calculator to determine a quadratic function \( h(t) \) that models the data.

10. Use the function from Item 9b to find the time when the rocket’s height is 450 ft. Verify that your result is the same as the data in the table.

11. After the parachute opened, how long did it take for the rocket to hit Earth?
Learning Targets:

- Solve quadratic equations.
- Interpret the solutions of a quadratic equation in a real-world context.

**SUGGESTED LEARNING STRATEGIES:** Create Representations, Predict and Confirm, Think-Pair-Share, Discussion Groups

1. The total time that the *Spirit* was in the air after the engine burned out is determined by finding the values of $t$ that make $h(t) = 0$.
   a. Rewrite the equation identified in Item 3a in Lesson 33-1 and set it equal to 0.
   b. Completely factor the equation.
   c. **Make use of structure.** Identify and use the appropriate property to find the time that the *Spirit* took to strike Earth after the engine burned out.

2. Draw a horizontal line on the graph in Item 1 in Lesson 33-1 to indicate a height of 544 ft above Earth. Estimate the approximate time(s) that the *Spirit* was 544 ft above Earth.
Lesson 33-2
Interpreting Solutions of Quadratic Equations

3. The time(s) that the Spirit was 544 ft above Earth can be determined exactly by finding the values of \( t \) that make \( h(t) = 544 \).
   a. Rewrite the equation from Item 3a in Lesson 33-1 and set it equal to 544.

   b. Is the method of factoring effective for solving this equation? Justify your response.

   c. Is the quadratic formula effective for solving this problem? Justify your response.

   d. Determine the time(s) that the rocket was 544 ft above Earth. Round your answer to the nearest thousandth of a second. Verify that this solution is reasonable compared to the estimated times from the graph in Item 2.

   e. Attend to precision. Explain why it is more appropriate in this context to round to the thousandths place rather than to use the exact answer or an approximation to the nearest whole number.

4. Cooper could not see the Spirit when it was higher than 528 ft above Earth.
   a. Calculate the values of \( t \) for which \( h(t) = 528 \).

   b. Reason quantitatively. Write an inequality to represent the values of \( t \) for which the rocket was not within Cooper’s sight.
Lesson 33-2
Interpreting Solutions of Quadratic Equations

Check Your Understanding

The path of a rocket is modeled by \( h(t) = -16t^2 + 45t + 220 \), where \( h \) is the height in feet and \( t \) is the time in seconds.

5. Determine the time \( t \) when the rocket was on the ground. Round to the nearest thousandth.

6. Identify the times, \( t \), when the height \( h(t) \) was greater than 220 ft.

LESSON 33-2 PRACTICE

Solve the quadratic equations. Round to the nearest thousandth.

7. \(-16t^2 + 100t + 316 = 0\)

8. \(-16t^2 + 100t + 316 = 100\)

Make sense of problems. For Items 9–12, use the function \( h(t) = -16t^2 + 8t + 30 \), which represents the height of a diver above the surface of a swimming pool \( t \) seconds after she dives.

9. The diver begins her dive on a platform. What is the height of the platform above the surface of the swimming pool? How do you know?

10. At what time does the diver reach her maximum height? What is the maximum height?

11. How long does it take the diver to reach the water? Round to the nearest thousandth.

12. Determine the times at which the diver is at a height greater than 20 ft. Explain how you arrived at your solution.
ACTIVITY 33 PRACTICE
Write your answers on notebook paper. Show your work.

Lesson 33-1
A model rocket is launched from the ground. Its height at different times after the launch is recorded in the table below. Use the table for Items 1–7.

<table>
<thead>
<tr>
<th>Time Since Launch of Rocket (s)</th>
<th>Height of Rocket (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>144</td>
</tr>
<tr>
<td>2</td>
<td>256</td>
</tr>
<tr>
<td>3</td>
<td>336</td>
</tr>
<tr>
<td>4</td>
<td>384</td>
</tr>
<tr>
<td>5</td>
<td>400</td>
</tr>
<tr>
<td>6</td>
<td>384</td>
</tr>
<tr>
<td>7</td>
<td>336</td>
</tr>
</tbody>
</table>

1. Are the data in the table linear? Justify your response.

2. Use a graphing calculator to determine a quadratic function \( h(t) \) for the data.

3. Identify a reasonable domain for the function within this context.

4. How high is the rocket after 8 seconds?

5. After how many seconds does the rocket come back to the ground?

6. What is the maximum height the rocket reached?

7. a. At what time(s) \( t \) will the height of the rocket be equal to 300 ft?

   b. How many times did you find in Part (a)? Explain why this makes sense in the context of the problem.

8. Which of the following functions best models the data in the table below?

   \[
   \begin{array}{c|c}
   x & f(x) \\
   \hline
   1 & 29 \\
   2 & 66 \\
   3 & 101 \\
   4 & 134 \\
   5 & 165 \\
   6 & 194 \\
   \end{array}
   \]

   A. \( f(x) = x + 28 \)
   B. \( f(x) = -x^2 + 40x - 10 \)
   C. \( f(x) = -16x^2 + 10x + 100 \)
   D. \( f(x) = -|x + 3| + 42 \)

9. As part of a fireworks display, a pyrotechnics team launches a fireworks shell from a platform and collects the following data about the shell's height.

<table>
<thead>
<tr>
<th>Time Since Launch of Shell (s)</th>
<th>Height of Shell (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>68</td>
</tr>
<tr>
<td>2</td>
<td>100</td>
</tr>
<tr>
<td>3</td>
<td>100</td>
</tr>
<tr>
<td>4</td>
<td>68</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
</tr>
</tbody>
</table>

Which of the following is a true statement about this situation?
A. The launch platform is 68 ft above the ground.
B. The maximum height of the shell is 100 ft.
C. The shell hits the ground after 6 seconds.
D. The shell starts to descend 2.5 seconds after it is launched.
Lesson 33-2
A soccer player passes the ball to a teammate, and the teammate kicks the ball. The function \( h(t) = -16t^2 + 14t + 4 \) represents the height of the ball, in feet, \( t \) seconds after it is kicked. Use this information for Items 10–16.

10. What is the height of the ball at the moment it is kicked? Justify your answer.

11. Graph the function.

12. Calculate the time at which the ball reaches its maximum height.

13. What is the maximum height of the ball?

14. Assuming no one touches the ball after it is kicked, determine the time when the ball falls to the ground.

15. Identify a reasonable domain and range for the function.

16. Determine the times when the ball is higher than 6 ft. Explain how you arrived at your solution.

Casey is standing on the roof of a building. She tosses a ball into the air so that her friend Leon, who is standing on the sidewalk, can catch it. The function \( y = -16x^2 + 32x + 80 \) models the height of the ball in feet, where \( x \) is the time in seconds. Use this information for Items 17–20.

17. Leon lets the ball hit the sidewalk. Determine the total time the ball is in the air until it hits the sidewalk.

18. Is the ball ever at a height of 100 ft? Justify your answer.

19. How many times is the ball at a height of exactly 92 ft?
   A. never \hspace{1cm} B. one time \hspace{1cm} C. two times \hspace{1cm} D. three times

20. Casey solves the equation shown below. What does the solution of the equation represent?
   \[ 10 = -16x^2 + 32x + 80 \]
   A. The height of the ball after 10 seconds \hspace{1cm} B. The time when the ball is at a height of 10 ft
   C. The time when the ball has traveled a total distance of 10 ft \hspace{1cm} D. The time it takes the ball to rise vertically 10 ft from the rooftop

The function \( h(t) = -16t^2 + 50t + k \), where \( k > 0 \), gives the height, in feet, of a marble \( t \) seconds after it is shot into the air from a slingshot. Determine whether each statement is always, sometimes, or never true.

21. The initial height of the marble is \( k \) feet.

22. There is some value of \( t \) for which the height of the marble is 0.

23. The graph of the function is a straight line.

24. The marble reaches a height of 50 ft.

25. The marble reaches a height of 65 ft.

26. The maximum height of the marble occurs at \( t = 1 \) second.

MATHEMATICAL PRACTICES
Reason Abstractly and Quantitatively

27. Why do you think quadratic functions are used to model free-fall motion instead of linear functions?
Every fall the Physics Club hosts an annual egg-drop contest. The goal of the egg-drop contest is to construct an egg-protecting package capable of providing a safe landing upon falling from a fifth-floor window.

During the egg-drop contest, each contestant drops an egg about 64 ft to a target placed at the foot of a building. The area of the target is about 10 square feet. Points are given for targeting, egg survival, and time to reach the target.

Colin wanted to win the egg-drop contest, so he tested one of his models with three different ways of dropping the package. These equations represent each method.

<table>
<thead>
<tr>
<th>Method</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Method A</td>
<td>$h(t) = -16t^2 + 64$</td>
</tr>
<tr>
<td>Method B</td>
<td>$h(t) = -16t^2 - 8t + 64$</td>
</tr>
<tr>
<td>Method C</td>
<td>$h(t) = -16t^2 - 48t + 64$</td>
</tr>
</tbody>
</table>

1. Quadratic equations can be solved by using square roots, by factoring, or by using the quadratic formula. Solve the three equations above to find $t$ when $h(t) = 0$. Use a different solution method for each equation. Show your work, and explain your reasoning for choosing the method you used.

2. Colin found that the egg would not break if it took longer than 1.5 seconds to hit the ground. Which method(s)—A, B, or C—will result in the egg not breaking?

3. Colin tried another method. This time he recorded the height of the egg at different times after it was dropped, as shown in the table below.

<table>
<thead>
<tr>
<th>Elapsed Time Since Drop of Egg (s)</th>
<th>Height of Egg (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>64</td>
</tr>
<tr>
<td>0.25</td>
<td>62</td>
</tr>
<tr>
<td>0.5</td>
<td>58</td>
</tr>
<tr>
<td>0.75</td>
<td>52</td>
</tr>
<tr>
<td>1.00</td>
<td>44</td>
</tr>
<tr>
<td>1.25</td>
<td>34</td>
</tr>
</tbody>
</table>

a. Use a graphing calculator to determine a function $h(t)$ that models the quadratic data.

b. After how many seconds will the egg hit the ground? Show the work that justifies your answer mathematically.

c. At what time, $t$, will the egg be 47 feet above the ground? Show the work that justifies your answer mathematically.
## Scoring Guide

The solution demonstrates the following characteristics:

<table>
<thead>
<tr>
<th>Mathematics Knowledge and Thinking (Item 1)</th>
<th>Exemplary</th>
<th>Proficient</th>
<th>Emerging</th>
<th>Incomplete</th>
</tr>
</thead>
<tbody>
<tr>
<td>Effective understanding of and accuracy in solving quadratic equations</td>
<td>Adequate understanding of how to solve quadratic equations, leading to solutions that are usually correct</td>
<td>Difficulty solving quadratic equations</td>
<td>Inaccurate or incomplete understanding of how to solve quadratic equations</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Problem Solving (Item 2)</th>
<th>Exemplary</th>
<th>Proficient</th>
<th>Emerging</th>
<th>Incomplete</th>
</tr>
</thead>
<tbody>
<tr>
<td>Appropriate and efficient strategy that results in a correct answer</td>
<td>Strategy that may include unnecessary steps but results in a correct answer</td>
<td>Strategy that results in some incorrect answers</td>
<td>No clear strategy when solving problems</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mathematical Modeling/ Representations (Item 3)</th>
<th>Exemplary</th>
<th>Proficient</th>
<th>Emerging</th>
<th>Incomplete</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clear and accurate understanding of how to use technology to model real-world data and how to use a graph to solve a real-world problem</td>
<td>Some difficulty understanding how to use technology to model real-world data and/or how to use a graph to solve a real-world problem, but a correct answer is present</td>
<td>Partial understanding of how to use technology to model real-world data and/or how to use a graph to solve a real-world problem that results in some incorrect answers</td>
<td>Little or no understanding of how to use technology to model real-world data and/or how to use a graph to solve a real-world problem</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Reasoning and Communication (Items 1, 3b, 3c)</th>
<th>Exemplary</th>
<th>Proficient</th>
<th>Emerging</th>
<th>Incomplete</th>
</tr>
</thead>
<tbody>
<tr>
<td>Precise use of appropriate math terms and language to explain a choice of solution method</td>
<td>Adequate explanation of choice of solution method</td>
<td>Misleading or confusing explanation of choice of solution method</td>
<td>Incomplete or inaccurate explanation of choice of solution method</td>
<td></td>
</tr>
<tr>
<td>Clear and accurate use of mathematical work to justify an answer</td>
<td>Correct use of mathematical work to justify an answer</td>
<td>Partially correct justification of an answer using mathematical work</td>
<td>Incorrect or incomplete justification of an answer using mathematical work</td>
<td></td>
</tr>
</tbody>
</table>
Modeling with Functions
Photo App
Lesson 34-1 Constructing Models

Learning Targets:
- Construct linear, quadratic, and exponential models for data.
- Graph and interpret linear, quadratic, and exponential functions.

**SUGGESTED LEARNING STRATEGIES:** Discussion Groups, Look for a Pattern, Create Representations, Sharing and Responding, Construct an Argument

Jenna, Cheyenne, and Kim all use a photo app on their smartphones to share their photos online and to track how many people follow their photo stories. The people who choose to follow the girls’ photo stories can also stop following or “unsubscribe” at any time. After the first eight months, the data for each of the girl’s monthly number of followers was compiled and is shown in the table below.

<table>
<thead>
<tr>
<th>Months Since Account Was Opened</th>
<th>Jenna’s Total Followers</th>
<th>Cheyenne’s Total Followers</th>
<th>Kim’s Total Followers</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>37</td>
<td>2</td>
<td>16</td>
</tr>
<tr>
<td>2</td>
<td>42</td>
<td>5</td>
<td>29</td>
</tr>
<tr>
<td>3</td>
<td>46</td>
<td>9</td>
<td>38</td>
</tr>
<tr>
<td>4</td>
<td>52</td>
<td>16</td>
<td>42</td>
</tr>
<tr>
<td>5</td>
<td>56</td>
<td>33</td>
<td>43</td>
</tr>
<tr>
<td>6</td>
<td>63</td>
<td>65</td>
<td>39</td>
</tr>
<tr>
<td>7</td>
<td>66</td>
<td>128</td>
<td>31</td>
</tr>
<tr>
<td>8</td>
<td>72</td>
<td>251</td>
<td>20</td>
</tr>
</tbody>
</table>

1. Describe any patterns you observe in the table. As you share your ideas with your group, be sure to use mathematical terms and academic vocabulary precisely. Make notes to help you remember the meaning of new words and how they are used to describe mathematical concepts.
2. **Reason quantitatively.** Use the terms *linear*, *quadratic*, or *exponential* to identify the type of function that would best model each girl's total number of followers during the first eight months.
   a. Jenna:

   b. Cheyenne:

   c. Kim:

3. Use the regression function of a graphing calculator to write a function that could be used to model each girl’s total number of followers over the first eight months.
   a. Jenna:

   b. Cheyenne:

   c. Kim:

4. Approximately how many followers does Jenna gain each month? Justify your response.

5. **Critique the reasoning of others.** Cheyenne tells the other girls she thinks her following is almost doubling each month. Is she correct? Justify your response using Cheyenne's function or the data in the table.

6. **Model with mathematics.** Use the functions from Item 3 to create graphs to represent each of the girl’s total number of followers over the first eight months. Label at least three points on each graph.
a. Describe the similarities and differences between the reasonable domains and ranges of each of the functions represented by the graphs.

b. Identify the maximum values, if they exist, of each of the functions represented by the graphs.
Check Your Understanding

7. The total number of songs that Ping has downloaded since he joined an online music club is shown in the table below. Use the regression function of a graphing calculator to write a function that best models the data.

<table>
<thead>
<tr>
<th>Days Since Joining</th>
<th>Total Songs Downloaded</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td>28</td>
</tr>
<tr>
<td>5</td>
<td>81</td>
</tr>
</tbody>
</table>

8. Use your function from Item 7 to describe how the total number of songs that Ping has downloaded is changing each day.
Lesson 34-1
Constructing Models

LESSON 34-1 PRACTICE

Alice and Ahmad have started an in-home pet grooming business. They each recorded the number of clients they visited each month for the first seven months. Their data are shown in the table below.

<table>
<thead>
<tr>
<th>Months in Business</th>
<th>Number of Alice’s Clients</th>
<th>Number of Ahmad’s Clients</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>11</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>13</td>
<td>8</td>
</tr>
<tr>
<td>6</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>7</td>
<td>18</td>
<td>32</td>
</tr>
</tbody>
</table>

9. Examine the data in the table to determine the type of function that would best model the number of each groomer’s clients.

10. Use the regression function of a graphing calculator to find a function to represent the number of Alice’s clients.

11. Use the regression function of a graphing calculator to find a function to represent the number of Ahmad’s clients.

12. Graph the functions from Items 10 and 11 on separate graphs.

13. **Attend to precision.** Describe the similarities and differences between the two functions. Compare and contrast the following features:
   a. domain and range
   b. maximum and minimum values
   c. rate of increase per day
   d. groomer with the most clients during different months
Learning Targets:

- Identify characteristics of linear, quadratic, and exponential functions.
- Compare linear, quadratic, and exponential functions.

**SUGGESTED LEARNING STRATEGIES:** Visualization, Discussion Groups, Construct an Argument, Create Representations

1. Rewrite the functions you found for Jenna, Kim, and Cheyenne in Lesson 34-1.
   
   Jenna:
   
   Cheyenne:
   
   Kim:

2. Which of the three girls—Jenna, Kim, or Cheyenne—has a constant rate of change in her number of followers per month? Explain.

3. Which girl had the greatest number of followers initially? Justify your response using both the functions and the graphs.

4. Did any of the girls experience followers who “unsubscribed” from, or stopped following, their photo story? Explain how you know.

5. Which girl’s photo story gained the most followers over the eight months? Justify your response using the functions or the graphs.

6. **Critique the reasoning of others.** Kim states that even if the number of Jenna’s followers had grown twice as quickly as it did, Cheyenne’s followers would still eventually outnumber Jenna’s followers. Is this assumption reasonable? Justify your response.
Lesson 34-2
Comparing Models

7. Write a new function that describes the total number of Jenna’s and Kim’s followers combined.

8. Determine the reasonable domain and range, as well as any maximum or minimum values, of the function you wrote in Item 7.

9. **Construct viable arguments.** Will the number of Cheyenne’s followers ever exceed the total number of Jenna and Kim’s followers combined? Justify your response.

Check Your Understanding

Write the letter of the description that matches the given function.

10. \( f(x) = -x^2 - 4x - 3 \)  
    A. has a constant rate of change

11. \( f(x) = 4x - 3 \)  
    B. has a maximum value

12. \( f(x) = 3(4)^x \)  
    C. increases very quickly at an ever increasing rate
LESSON 34-2 PRACTICE
The graphs show the number of times two online retailers—Roberto’s Books and Tyler’s Time to Read—recommended the bestselling book *My Story* to their customers after *x* days. Use the graphs for Items 13–16.

13. Who had given more recommendations of *My Story* after three days?
14. Who recommended *My Story* the same number of times each day? How many times was it recommended each day?
15. If this model continues, Roberto’s Books will have recommended *My Story* approximately 1465 times after one year. Is this a reasonable amount? Justify your response.
16. Will the number of times Tyler’s Time to Read recommends *My Story* ever exceed the number of times Roberto’s Books recommends *My Story*? Explain your reasoning.

17. **Make use of structure.** The total number of times that Roberto’s Books recommended the bestseller *A Fatal Memory* after *x* days is modeled by the function \( f(x) = 3x + 2 \). The total number of times that Penny’s Place recommended the same book after *x* days is modeled by the function \( g(x) = -4x^2 + 12 \). Write a function to model the combined number of times that Roberto’s Books and Penny’s Place recommended *A Fatal Memory*. 

**MATH TIP**
You can combine algebraic representations for two different functions by adding, subtracting, multiplying, or dividing the expressions.
Learning Targets:
• Compare piecewise-defined, linear, quadratic, and exponential functions.
• Write a verbal description that matches a given graph.

SUGGESTED LEARNING STRATEGIES: Look for a Pattern, Create Representations, Critique Reasoning, Think-Pair-Share, Marking the Text

Rosa begins using the photo app to create a photo story and track her number of followers. The table below shows her results.

<table>
<thead>
<tr>
<th>Months Since Account Was Started</th>
<th>Rosa’s Total Followers</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>25</td>
</tr>
<tr>
<td>2</td>
<td>25</td>
</tr>
<tr>
<td>3</td>
<td>25</td>
</tr>
<tr>
<td>4</td>
<td>25</td>
</tr>
<tr>
<td>5</td>
<td>50</td>
</tr>
<tr>
<td>6</td>
<td>50</td>
</tr>
<tr>
<td>7</td>
<td>50</td>
</tr>
<tr>
<td>8</td>
<td>50</td>
</tr>
</tbody>
</table>

1. Describe any patterns you observe in the table.

2. Graph the data in the table.

3. Write a function to model Rosa’s number of followers for the first four months.

4. Write a function to model Rosa’s number of followers for months five through eight.
5. Write a **piecewise-defined function** to represent the number of followers Rosa has in any given month.

6. **Critique the reasoning of others.** Rosa says that her total number of followers is changing at a constant rate of 25 followers per month for the first four months. Is Rosa’s statement correct? Explain your reasoning.

Juanita has recorded the number of followers for her latest photo story over the last seven days. She finds that the function $f(x) = -5|x - 4| + 45$ represents the number of followers after $x$ days.

7. Complete the table for the number of followers each day.

<table>
<thead>
<tr>
<th>Days Since Photo Story Was Posted</th>
<th>Number of Followers</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
</tr>
</tbody>
</table>

8. Graph the function that models Juanita’s data.
Lesson 34-3
Extending Models

9. Determine the reasonable domain and range, as well as the maximum and minimum values of the function within the context of the problem.

10. **Reason abstractly.** Describe the similarities and differences between the function \( f(x) = -5|x - 4| + 45 \) and a quadratic function.

11. The graphs below show the number of followers for two photo stories over one week. Describe how the number of followers changed over time.
   a. 
   ![Graph a]
   
   b. 
   ![Graph b]
LESSON 34-3 PRACTICE
Make sense of problems. The graphs show the number of raffle tickets Mike and Ryan each sold to raise money for a school group.

14. For which days did the number of raffle tickets that Mike sold increase at a constant rate?
15. For which days did the number of raffle tickets that Ryan sold not increase?
16. Describe any patterns you see in the number of tickets that Mike sold over the seven days.
17. Compare and contrast any patterns you observe in the number of tickets that Mike sold and the number of tickets Ryan sold over the seven days.

Check Your Understanding
Use the graph for Items 12 and 13.

12. During which time period was the number of bacteria constant?
13. Describe the type of change in the number of bacteria for hours four through seven.

LESSON 34-3 PRACTICE
Make sense of problems. The graphs show the number of raffle tickets Mike and Ryan each sold to raise money for a school group.

14. For which days did the number of raffle tickets that Mike sold increase at a constant rate?
15. For which days did the number of raffle tickets that Ryan sold not increase?
16. Describe any patterns you see in the number of tickets that Mike sold over the seven days.
17. Compare and contrast any patterns you observe in the number of tickets that Mike sold and the number of tickets Ryan sold over the seven days.
Li and Alfonso have both opened accounts with an online music store. The data in the table below show the total number of songs each of them has downloaded since opening his account. Use the table for Items 1–12.

<table>
<thead>
<tr>
<th>Days Since Account Opened</th>
<th>Total Number of Songs Downloaded by Li</th>
<th>Total Number of Songs Downloaded by Alfonso</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>15</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>26</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
<td>34</td>
</tr>
<tr>
<td>5</td>
<td>16</td>
<td>45</td>
</tr>
<tr>
<td>6</td>
<td>33</td>
<td>56</td>
</tr>
<tr>
<td>7</td>
<td>65</td>
<td>67</td>
</tr>
</tbody>
</table>

**Lesson 34-1**

1. What type of function would best model the number of songs that Li has downloaded after $x$ days?
   - A. linear
   - B. quadratic
   - C. exponential
   - D. absolute value

2. What type of function would best model the number of songs that Alfonso has downloaded after $x$ days?
   - A. linear
   - B. quadratic
   - C. exponential
   - D. absolute value

**Lesson 34-2**

6. How would the number of songs downloaded by Alfonso change if the rate of change of Alfonso’s downloads remained the same, but he had not downloaded any songs on day 1?

7. Describe the similarities and differences between the rates of change in the number of songs downloaded by the two boys.

8. If the models continue to represent the number of songs downloaded, who do you predict will have downloaded more songs after 30 days? Explain your reasoning.

9. If the rate of change of the number of songs downloaded by Alfonso tripled, how many songs will he have downloaded after 30 days? Is this a reasonable number?

10. How many songs will Li have downloaded after 30 days if his model continues? Is this a reasonable number?
Lesson 34-3
Caily opened an account at the same time with the same online music store. The following piecewise function represents the total number of songs she has downloaded over the first \( x \) days.

\[
f(x) = \begin{cases} 
10 & \text{when } 1 \leq x < 15 \\
30 & \text{when } 15 \leq x < 25 \\
45 & \text{when } 25 \leq x \leq 30
\end{cases}
\]

Use this function for Items 11 and 12.

11. Describe the number of songs Caily has downloaded during this time.

12. Describe the difference between the number of songs downloaded by Caily and by Alfonso over the first 30 days.

13. The functions \( g(x) = 10x + 2 \) and \( h(x) = -0.5x^2 + 4x + 30 \) represent the total numbers of two different types of fish in a pond over \( x \) weeks. Write a function \( k(x) \) that represents the combined number of fish during the same period of time.

14. Graph the function \( k(x) \) from Item 13. What is the maximum value of the function?

MATHEMATICAL PRACTICES
Construct Viable Arguments and Critique the Reasoning of Others

15. The graph shows the number of “likes” a blogger received for a blog post \( t \) days after it was posted.

The blogger believes that there may have been something wrong with how the “likes” were recorded between days 10 and 20. Why might she believe this?
Learning Targets:
• Write a function to model a real-world situation.
• Solve a system of equations by graphing.

SUGGESTED LEARNING STRATEGIES: Marking the Text, Create Representations, Sharing and Responding, Discussion Groups, Visualization

Professor Hearst is studying different types of bacteria in order to determine new ways to prevent their population overgrowth. Each bacterium in the first culture that she examines divides to produce another bacterium once each minute. In the second culture, she observes that the number of bacteria increases by 10 bacteria each second.

1. Each population began with 10 bacteria. Complete the table for the population of each bacteria sample.

<table>
<thead>
<tr>
<th>Elapsed Time (minutes)</th>
<th>Population of Sample A</th>
<th>Population of Sample B</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. **Reason quantitatively.** Describe the type of function that would best model each population.

Bacteria can be harmful or helpful to other organisms, including humans. Bacteria are commonly used for food fermenting, waste processing, and pest control.
3. Write a function \( A(t) \) to model the number of bacteria present in Sample A after \( t \) minutes.

4. Write a function \( B(t) \) to model the number of bacteria present in Sample B after \( t \) minutes.

5. **Use appropriate tools strategically.** Use a graphing calculator to graph \( A(t) \) and \( B(t) \) on the same coordinate plane.
   a. Sketch the graph below and label several points on each graph.

   ![Graph with labeled points]

   b. Determine the points of intersection of the two graphs. Round non-integer values to the nearest tenth.

   c. **Make use of structure.** What do the points of intersection indicate about the two graphs? Explain.

   d. Interpret the meaning of the points of intersection within the context of the bacteria samples.

Lesson 35-1
Solving a System Graphically

The solutions you found in Item 5b are solutions to the nonlinear system of equations $A(t) = 10(2)^t$ and $B(t) = 600t + 10$.

Just as with linear systems, you can solve nonlinear systems by graphing each equation and determining the intersection point(s).

7. Solve each system of equations by graphing.
   a. $y = 2x + 1$
      $y = x^2 + 1$

   b. $y = x - 1$
      $y = 3^x - 2$

   c. $y = 2(5)^x$
      $y = -x^2 + 2x - 2$
LESSON 35-1 PRACTICE

For Items 11–13, solve each system of equations by graphing.

11. \[ y = -2x + 4 \]
    \[ y = -x^2 + 3 \]

12. \[ y = x^2 + 5x - 3 \]
    \[ y = -x^2 + 5x + 1 \]

13. \[ y = -4x^2 - 1 \]
    \[ y = 4(0.5)^x \]

14. A nonlinear system contains one linear equation and one quadratic equation. The system has no solutions. Sketch a possible graph of this system.

15. Is it possible for a system with one linear equation and one exponential equation to have two solutions? If so, sketch a graph that could represent such a system. If not, explain why not.

16. Critique the reasoning of others. A population of 200 bacteria begins increasing at a constant rate of 100 bacteria per minute. Francis writes the function \( P(t) = 200(100)^t \), where \( t \) represents the time in minutes, to model this population. Fred disagrees. He writes the function \( P(t) = 200 + 100t \) to model this population. Who is correct? Justify your response.
Lesson 35-2
Solving a System Algebraically

Learning Targets:
- Write a system of equations to model a real-world situation.
- Solve a system of equations algebraically.

SUGGESTED LEARNING STRATEGIES: Create a Plan, Identify a Subtask, Construct an Argument, Close Reading, Note Taking, Visualization, Think-Pair-Share, Discussion Groups

Just as with linear systems, nonlinear systems of equations can be solved algebraically.

Example A

Solve the system of equations algebraically.

\[ y = -x + 3 \]
\[ y = x^2 - 2x - 3 \]

Step 1: The first equation is solved for \( y \).

Substitute the expression for \( y \) into the second equation.

\[ y = x^2 - 2x - 3 \]
\[ (-x + 3) = x^2 - 2x - 3 \]

Step 2: Solve the resulting equation using any method. In this case, solve by factoring.

\[ 0 = x^2 - x - 6 \]
\[ 0 = (x + 2)(x - 3) \]

Step 3: Apply the Zero Product Property.

\[ x + 2 = 0 \text{ or } x - 3 = 0 \]

Step 4: Solve each equation for \( x \).

\[ x = -2 \text{ or } x = 3 \]

Step 5: Calculate the corresponding \( y \)-values by substituting the \( x \)-values from Step 4 into one of the original equations.

When \( x = -2 \), \( y = (-2)^2 - 2(-2) - 3 = 5 \)
When \( x = 3 \), \( y = -(3) + 3 = 0 \)

Step 6: Check the solution by substituting into the other original equation.

When \( x = -2 \), \( y = 5 \)
When \( x = 3 \), \( y = 0 \)

Solution: \((-2, 5)\) and \((3, 0)\)

Try These A

Solve each system algebraically.

a. \[ y = -x + 2 \]
\[ y = x^2 - x + 2 \]

b. \[ y = 2x^2 - 7 \]
\[ y = 7x - 3 \]

c. \[ y = -x + 3 \]
\[ y = x^2 - 2x - 4 \]
1. Lauren solved the following system of equations algebraically and found two solutions.

\[ y = -x + 3 \]
\[ y = x^2 - 3x - 4 \]

Will solved the system by graphing and said that there is only one solution. Who is correct? Justify your response both algebraically and graphically.

2. **Model with mathematics.** Deshawn drops a ball from the 520-foot-high observation deck of a tower. The height of the ball in feet after \( t \) seconds is given by \( f(t) = -16t^2 + 520 \). At the moment the ball is dropped, Zoe begins traveling up the tower in an elevator that starts at the ground floor. The elevator travels at a rate of 12 feet per second. At what time will Zoe and the ball pass by each other?

   a. Write a function \( g(t) \) to model Zoe’s height above the ground after \( t \) seconds.

   b. Write a system of equations using the function modeling the height of the ball and the function you wrote in Part (a).

   c. Solve the system that you wrote in Part (b) algebraically. Round to the nearest hundredth, if necessary.
d. Interpret the meaning of the solution in the context of the problem. Does the solution you found in Part (c) make sense? Explain.

e. Determine the height at which Zoe and the ball pass by each other. Explain how you found your answer.

3. At the same moment that Deshawn drops the ball, Joey begins traveling down the tower in another elevator that starts at the observation deck. This elevator also travels at a rate of 12 feet per second.

a. Write a function $h(t)$ to model Joey's height above the ground after $t$ seconds. Explain any similarities or differences between this function and the function in Item 2a.

b. Solve the system of equations algebraically.

c. Interpret the solution in the context of the problem.

d. Construct viable arguments. Determine whether Joey or the ball reaches the ground first. Justify your response.
LESSON 35-2 PRACTICE

Solve each system of equations algebraically.

6. \[ y = 16x - 13 \quad 7. \ y = 5 \]
   \[ y = 4x^2 + 3 \quad y = -x^2 - x + 1 \]

8. \[ y = x \]
   \[ y = x^2 + 2x - 4 \]

9. Jessica has decided to solve a system of equations by graphing. Her graph is shown. Why might she prefer to solve this system algebraically?

10. Make sense of problems. A competitive diver dives from a 33-foot high diving board. The height of the diver in feet after \( t \) seconds is given by \( u(t) = -16t^2 + 4t + 33 \). At the moment the diver begins her dive, another diver begins climbing the diving board ladder at a rate of 2 feet per second. At what height above the pool deck do the two divers pass each other?
ACTIVITY 35 PRACTICE
Write your answers on notebook paper. Show your work.

Lesson 35-1

1. Which function models the size of a neighborhood that begins with one home and doubles in size every year?
   A. \( P(t) = t + 2 \)
   B. \( P(t) = 2t + 1 \)
   C. \( P(t) = 2(1)^t \)
   D. \( P(t) = (2)^t \)

2. Which function models the size of a neighborhood that begins with 4 homes and increases by 6 homes every year?
   A. \( P(t) = 4t + 6 \)
   B. \( P(t) = 6t + 4 \)
   C. \( P(t) = 6^t + 4 \)
   D. \( P(t) = 4(6)^t \)

3. When will the number of homes in Items 1 and 2 be equal?

For Items 4 and 5, sketch the graph of a system that matches the description. If no such system exists, write not possible.

4. The system contains a linear equation and an exponential equation. There are no solutions.

5. The system contains two exponential equations. There is one solution.

For Items 6–8, solve each system of equations by graphing.

6. \( y = -x^2 + 3 \)
   \( y = x^2 + 4 \)

7. \( y = 0.5x^2 + x - 2 \)
   \( y = x - 2 \)

8. \( y = 2x^2 + 5 \)
   \( y = -2x + 5 \)

9. Twin sisters Tamara and Sandra each receive $50 as a birthday present. Tamara puts her money into an account that pays 3% interest annually. The amount of money in Tamara’s account after \( x \) years is given by the function \( t(x) = 50(1.03)^x \).

Sandra puts her money into a checking account that does not pay interest, but she plans to deposit $50 per year into the account. The amount of money in Sandra’s account after \( x \) years is given by the function \( s(x) = 50x + 50 \).

Use a graphing calculator to graph this system of equations. When will Tamara and Sandra have an equal amount of money in their accounts?

10. Josie and Jamal sold granola bars as a fund raiser, and they each started with 128 granola bars to sell. Josie sold 30 granola bars every day. The number of granola bars that Josie sold after \( x \) days is given by the function \( y = -30x + 128 \).

Every day, Jamal sold half the number of the granola bars than he sold the day before. The number of granola bars that Jamal sold after \( x \) days is given by the function \( y = 128(0.5)^x \).

Use a graphing calculator to graph this system of equations. After how many days did Josie and Jamal have the same number of unsold granola bars?
For Items 11 and 12, write a system of equations to model the situation. Then solve the system by graphing.

11. A sample of bacteria starts with 2 bacteria and doubles every minute. Another sample starts with 4 bacteria and increases at a constant rate of 2 bacteria every minute. When will the populations be equal?

12. Jennie and James plan to save money by raking leaves. Jennie already has 1 penny. With each bag of leaves she rakes, she doubles the amount of money she has. James earns 10 cents per bag. When will Jennie have more money than James?

Lesson 35-2

For Items 13–17, solve each system of equations algebraically.

13. \[ y = 3x^2 - x - 2 \]
   \[ y = 2x + 3 \]

14. \[ y = x^2 - 81 \]
   \[ y = 18x - 161 \]

15. \[ y = x^2 + 4 \]
   \[ y = 4x \]

16. \[ y = 3x + 3 \]
   \[ y = x^2 + 3x + 2 \]

17. \[ y = 3x \]
   \[ y = 2x^2 \]

For Items 18 and 19, write a system of equations to model the scenarios. Then solve the system of equations algebraically.

18. Simone is driving at a rate of 60 mi/h on the highway. She passes Jethro just as he begins accelerating onto the highway from a complete stop. The distance that Jethro has traveled in feet after \( t \) seconds is given by the function \( f(t) = 5.5t^2 \). When will Jethro catch up to Simone? (Hint: Use the fact that 60 mi/h is equivalent to 88 ft/s to write a function that gives the distance Simone has traveled.)

19. Jermaine is playing soccer next to his apartment building. He kicks the ball such that the height of the ball in feet after \( t \) seconds is given by the function \( g(t) = -16t^2 + 48t + 2 \). At the same moment that he kicks the ball, Jermaine’s father begins descending in the elevator from his apartment at a rate of 5 ft/s. The apartment is 30 feet above the ground floor. At what time(s) are Jermaine’s father and the soccer ball at the same height?

Mathematical Practices
Construct Viable Arguments and Critique the Reasoning of Others

20. Rachel is solving systems of equations and has concluded that the quadratic formula is always an appropriate solution method when solving a nonlinear system algebraically. Do you agree with Rachel’s conclusion? Use examples to support your reasoning.
Emilio loves sports and sports memorabilia. He has collected many different items over the years, but his favorite items are a signed baseball card, the catcher’s mitt of his favorite catcher from 1979, and a vintage pennant from his favorite team. Emilio enjoys keeping track of how much his items are worth and has tracked their values for the last 10 years. The table below shows the values he has recorded thus far.

<table>
<thead>
<tr>
<th>Years Since Item Acquired</th>
<th>Signed Baseball Card</th>
<th>Catcher’s Mitt</th>
<th>Vintage Pennant</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$50.00</td>
<td>$30.00</td>
<td>$6.00</td>
</tr>
<tr>
<td>2</td>
<td>$53.00</td>
<td>$37.00</td>
<td>$13.00</td>
</tr>
<tr>
<td>3</td>
<td>$56.00</td>
<td>$45.00</td>
<td>$22.00</td>
</tr>
<tr>
<td>4</td>
<td>$59.00</td>
<td>$55.00</td>
<td>$33.00</td>
</tr>
<tr>
<td>5</td>
<td>$62.00</td>
<td>$68.00</td>
<td>$46.00</td>
</tr>
<tr>
<td>6</td>
<td>$65.00</td>
<td>$83.00</td>
<td>$61.00</td>
</tr>
<tr>
<td>7</td>
<td>$68.00</td>
<td>$101.00</td>
<td>$78.00</td>
</tr>
<tr>
<td>8</td>
<td>$71.00</td>
<td>$124.00</td>
<td>$97.00</td>
</tr>
<tr>
<td>9</td>
<td>$74.00</td>
<td>$152.00</td>
<td>$118.00</td>
</tr>
<tr>
<td>10</td>
<td>$77.00</td>
<td>$186.00</td>
<td>$141.00</td>
</tr>
</tbody>
</table>

1. Identify the type of function that can be used to represent the value of each of the items shown in the table. Use the regression functions of a graphing calculator to determine functions \( S(t) \), \( C(t) \), and \( P(t) \) that model the value of the signed baseball card, the catcher’s mitt, and the vintage pennant, respectively.

2. In addition to the items in the table, Emilio also has a baseball that he caught during his favorite game of all time. The function \( B(t) = -100|t - 5| + 1300 \) can be used to model the value of the ball. Graph this function and each of the other three functions on separate coordinate planes.

3. Identify the appropriate domain for each function. Justify your responses.

4. Which item reached the greatest value during the last 10 years? What is that maximum value? Justify your response.

5. Which item's value is changing at a constant rate? Support your response using a graph or the table.

6. Which item or items have a decreasing value? For what values of \( t \) do the function(s) decrease? Justify your response.

7. Which item's value is increasing the fastest? Explain your reasoning.

8. After how many years will the signed baseball card be worth more than the catcher's mitt? Use graphs to justify your response.

9. Create a system of equations to represent the relationship between the values of the signed baseball card and the vintage pennant. Solve the system algebraically. Interpret the solution within the context of the sports memorabilia. Describe how the values of the two items compare before and after the time represented by the system’s solution.
### Scoring Guide

The solution demonstrates the following characteristics:

#### Mathematics Knowledge and Thinking (Items 1–9)
- **Exemplary**
  - Clear and accurate understanding of piecewise-defined, linear, quadratic, and exponential functions and the key features of their graphs
  - Effective understanding of how to solve systems of equations graphically and algebraically

- **Proficient**
  - Largely correct understanding of piecewise-defined, linear, quadratic, and exponential functions and the key features of their graphs
  - Adequate understanding of how to solve systems of equations graphically and algebraically

- **Emerging**
  - Partial understanding of piecewise-defined, linear, quadratic, and exponential functions and the key features of their graphs
  - Some difficulty solving systems of equations graphically and/or algebraically

- **Incomplete**
  - Inaccurate or incomplete understanding of piecewise-defined, linear, quadratic, and exponential functions and the key features of their graphs
  - Little or no understanding of how to solve systems of equations graphically and/or algebraically

#### Problem Solving (Items 4, 8)
- **Exemplary**
  - Appropriate and efficient strategy that results in a correct answer

- **Proficient**
  - Strategy that may include unnecessary steps but results in a correct answer

- **Emerging**
  - Strategy that results in a partially incorrect answer

- **Incomplete**
  - No clear strategy when solving problems

#### Mathematical Modeling / Representations (Items 1–9)
- **Exemplary**
  - Clear and accurate understanding of how to find, graph, interpret, and compare regression functions that model real-world data
  - Effective understanding of how to write and interpret the solution of a system of equations that represents a real-world scenario

- **Proficient**
  - Largely correct understanding of how to find, graph, interpret, and compare regression functions that model real-world data
  - Adequate understanding of how to write and interpret the solution of a system of equations that represents a real-world scenario

- **Emerging**
  - Partial understanding of how to find, graph, interpret, and/or compare regression functions that model real-world data
  - Some difficulty writing and/or interpreting the solution of a system of equations that represents a real-world scenario

- **Incomplete**
  - Inaccurate or incomplete understanding of how to find, graph, interpret, and/or compare regression functions that model real-world data
  - Little or no understanding of how to write and/or interpret the solution of a system of equations that represents a real-world scenario

#### Reasoning and Communication (Items 3–9)
- **Exemplary**
  - Precise use of appropriate math terms and language to describe and compare characteristics of several functions

- **Proficient**
  - Adequate description and comparison of characteristics of several functions

- **Emerging**
  - Misleading or confusing description and comparison of characteristics of several functions

- **Incomplete**
  - Incomplete or inaccurate description and comparison of characteristics of several functions